Linear Programming
Essay – EKO 203
Lucie Pietroszová, LS 2003

Our firm manufactures 2 types of lamps: plastic and metallic. The plastic lamp costs 600 CZK and the metallic one costs 900 CZK. It takes 1,5 kg of plastic material, 0,5 kg of metal and 1 hour of machine labor to produce one plastic lamp. To produce one metallic lamp it takes 1 kg of plastic material, 2 kg of metal and 1 hour of machine labor. We have at disposal 1500 kg of plastic material, 2000 kg of metal and 1500 machine hours each month. Management wants to maximize monthly total revenue.

	Plastic Lamp	Metal Lamp	Amount at disposal
Price	600	900	
Plastic	1,5	1	1500
Metal	0,5	2	2000
Machine hours	1	1	1500

Decision Variables

x_1 = number of plastic lamps produced each month,
x_2 = number of metal lamps produced each month

Objective Function

We wants to maximize total revenue (price*amount of production) each month.
Total revenue (TR) = $600x_1 + 900x_2$ .... MAX

Constraints

There are three restrictions:
Each month the firm has only 1500 available kilograms of plastic material, 2000 kg of metal and 1500 available hours of machine labor.

$$1,5x_1 + x_2 \leq 1500$$
$$0,5x_1 + 2x_2 \leq 2000$$
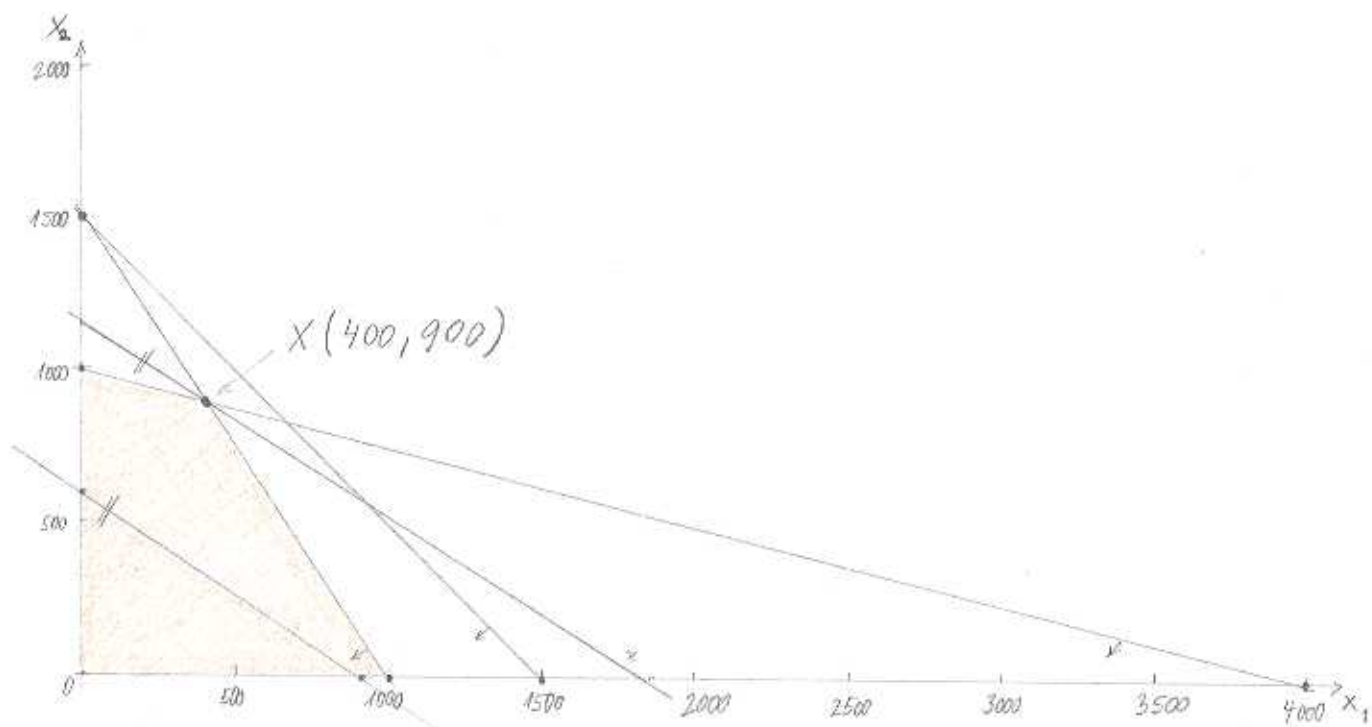$$x_1 + x_2 \leq 1500$$

Nonnegativity constraints:
$x_1, x_2 \geq 0$

Summarized Mathematical Model:

$z = 600x_1 + 900x_2 \dots\dots \max$
$1,5x_1 + x_2 \leq 1500$
$0,5x_1 + 2x_2 \leq 2000$
$\quad x_1 + x_2 \leq 1500$
$\quad x_1, x_2 \geq 0$

Graphical Solution

$X(400, 900)$

Optimal Solution

$1,5x_1 + x_2 = 1500$
$0,5x_1 + 2x_2 = 2000$
$\quad \dots x_2 = 1500 - 1,5x_1$

$0,5x_1 + 2(1500 - 1,5x_1) = 2000$
$2,5x_1 = 1000$
$x_1 = 400$
$x_2 = 900$

Optimal Solution: X (400, 900)
$z = 1\ 050\ 000$

If the firm wants to maximize total revenue, it has to produce 400 plastic lamps and 900 metal lamps each month. The total revenue will be 1 050 000 CZK each month.

Slack/Surplus Variable

$1{,}5*400 + 900 = 1500$

Left-hand side exactly equals the right-hand side. The constraint is binding. It means that all plastic material is used for the optimal production.

$0{,}5*400 + 2*900 = 2000$

The constraint is binding too. No metal remains, so all the available metal is used for the optimal production.

$400 + 900 \neq 1500$

The constraint is nonbinding. There is a difference between the right-hand side and the left-hand side of the constraint. The **slack variable** in machine hour constraint is 200 hours.