ŠKODA AUTO University

Modeling of Production and Logistics Systems

Lectures

Jan Fábry

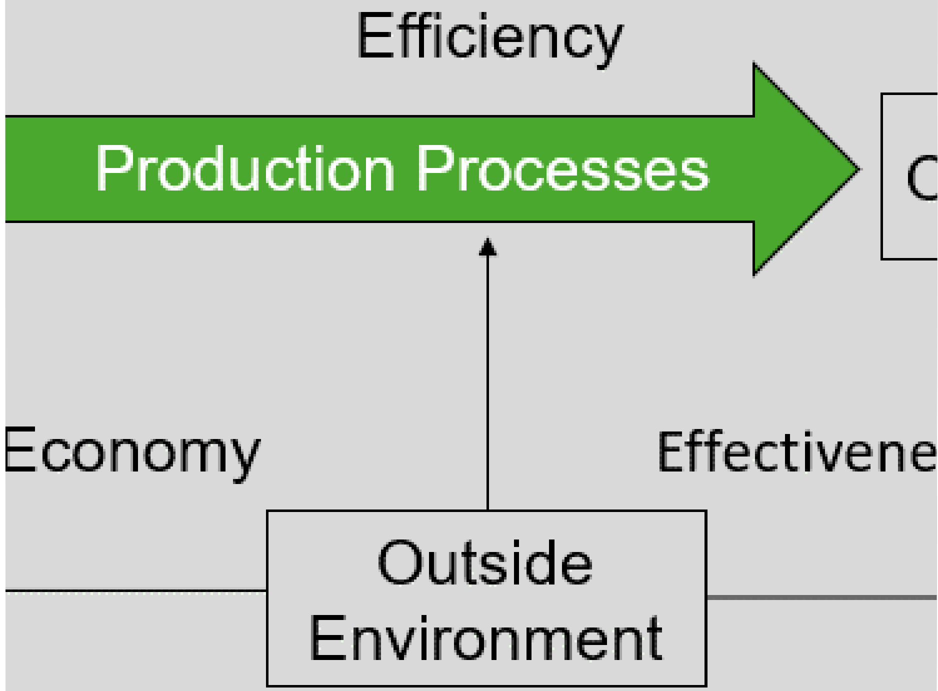
5/13/2022

Contents

ŠKODA AUTO University

1

Basic terminology of production systems



Basic terminology of production systems

Basic model of production system

- 3E strategy
  - Economy
  - Efficiency
  - Effectiveness
- Efficiency

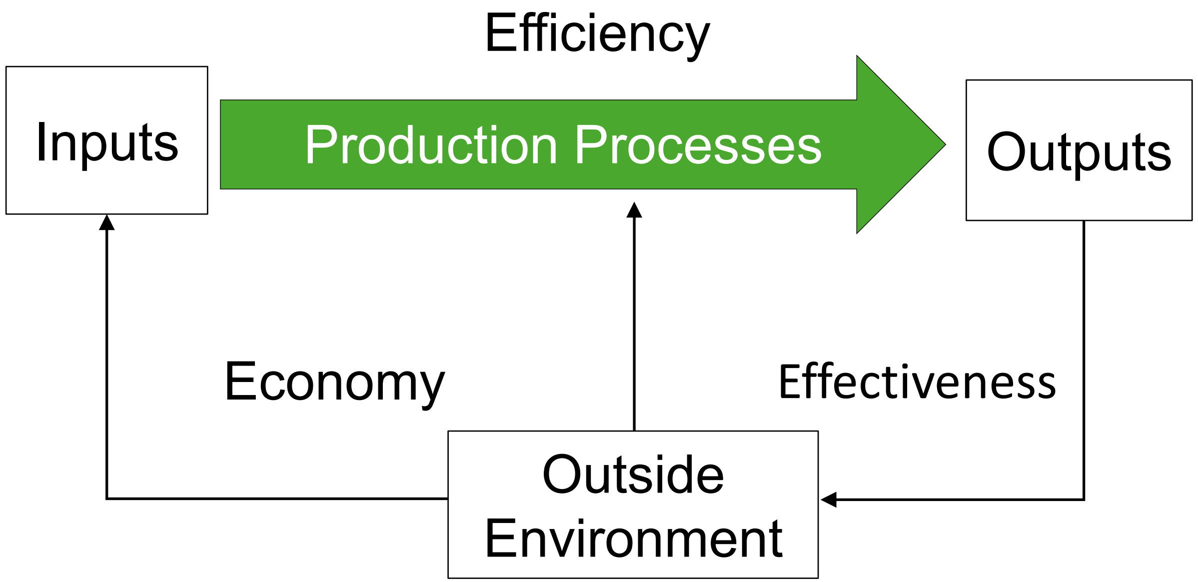
$z = TR - TC$

$TR = pq$

$TC = FC + VC$

$VC = vq$

$z = pq - (FC + vq) = (p - v)q - FC$



Basic terminology of production systems

Basic model of production system

- STEEP analysis
    - S = social
    - T = technological
    - E = economic
    - E = ecological
    - P = political

Influence of external factors on the production system

- SWOT analysis
    - S = strenghts
    - W = weeknesses
    - O = opportunities
    - T = threats

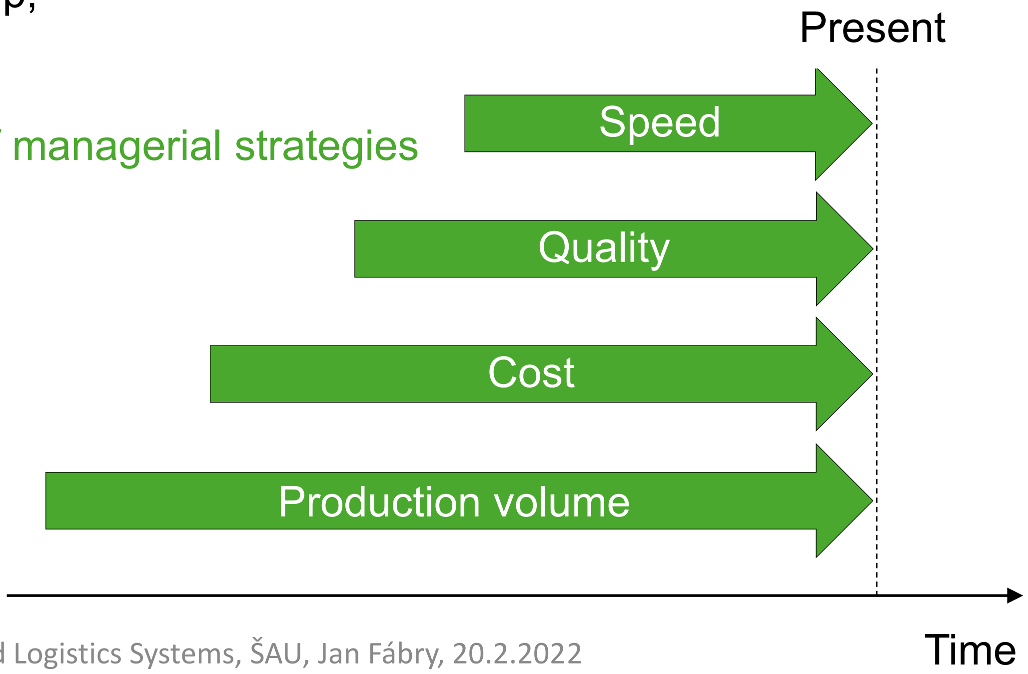
Existing situation of the company in relation to its outside environment

Basic terminology of production systems

ŠKODA AUTO University

Management concepts and types of production systems

- Main functions of management
  - planning,
  - decision making,
  - organizing,
  - leadership,
  - control.
- Development of managerial strategies



Basic terminology of production systems

ŠKODA AUTO University

Management concepts and types of production systems

- Basic managerial conceptions
  - **JIT (Just-in-Time)** – the required quantity at the required time.
  - **Lean Production System** – elimination of useless processes, quality improvement.
  - **TOC (Theory of Constraints)** – elimination of bottlenecks, optimising throughput.
  - **TQM (Total Quality Management)** – comprehensive quality management, benchmarking.
  - **QRM (Quick Response Manufacturing)** – speeding up processes, shortening delivery times.
  - **TEI (Total Employee Involvement)** – employee involvement in quality improvement.
  - **BPR (Business Process Reingeneering)** – changing business processes to improve the competitiveness of the company.
  - **Agile Manufacturing** – the aim is to respond quickly to customer needs.
  - **SCM (Supply Chain Management)** – optimising supply chains.

Basic terminology of production systems

ŠKODA AUTO University

Management concepts and types of production systems

- Types of production systems
  - **Type of operations**
    - production operations – transformation of inputs into outputs, easier measurability of quality,
    - non-production operations – providing services, closer contact with customers.
  - **Type of product**
    - production companies,
    - service companies,
    - companies providing products and services.
  - **Type of production**
    - production to stock – stock based on an estimate of expected orders,
    - production to order (contract) – special customer requirements,
    - assembly to order (contract) – a combination of both, the product is assembled according to a special order from stocked components.

Basic terminology of production systems

ŠKODA AUTO University

Management concepts and types of production systems

- Types of production systems
  - **Type of production**
    - job production - low repeatability, high variability,
    - batch production - the same products are produced consecutively in a certain quantity (in a series),
    - mass production - high level of repeatability, low level of variability.

Basic terminology of production systems

ŠKODA AUTO University

Forecasts

- Time
  - short range forcasts (up to 1 year),
  - medium range forcasts (1-5 years),
  - long range forecasts (over 5 years).
- Focus
  - technological forecasts – estimation of technical and technological progress,
  - economic forecasts – estimation of changes in values of basic economic indicators,
  - forecasts of future demand.

Basic terminology of production systems

ŠKODA AUTO University

Phases of production systems analysis

- Designing production systems.
- Management of production systems.
- Measurement of production systems performance.
- Improving the performance of production systems.

Basic terminology of production systems

ŠKODA AUTO University

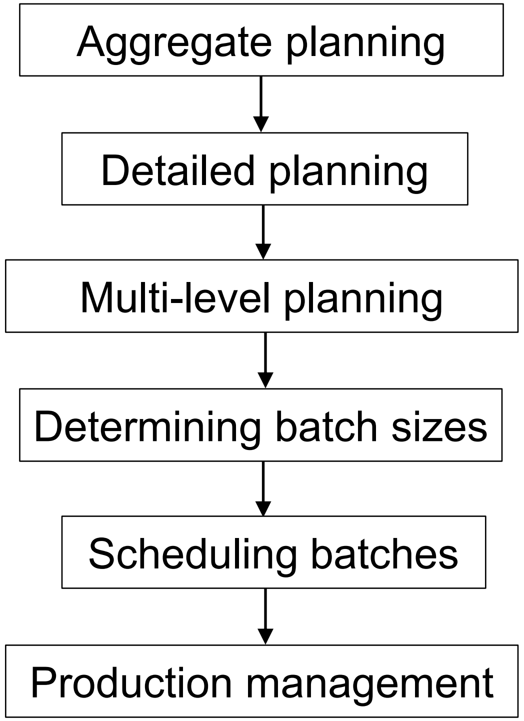
Phases of production systems analysis

- Designing production systems
  - **Product designing** – what will be produced?
  - **Process designing** – how will it be produced?
    - selection of technology,
    - capacity planning
      - long-term planning – high capital expenditures,
      - short-term planning – efficient use of existing equipment,
    - the location of the facilities – where the facilities will be placed,
    - the schedule of the facilities – the selection of the way of processing the products on the facilities
      - fixed position of the product (house, bridge, aircraft),
      - schedule by process (contract production),
      - schedule by product (batch production),
      - group (cell) schedule – grouping of facilities with similar operations.

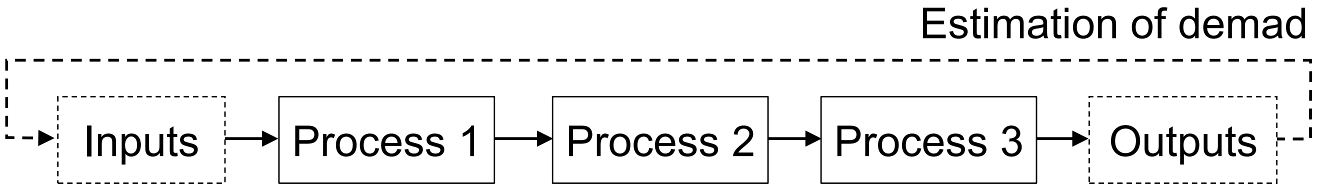
Basic terminology of production systems

Phases of production systems analysis

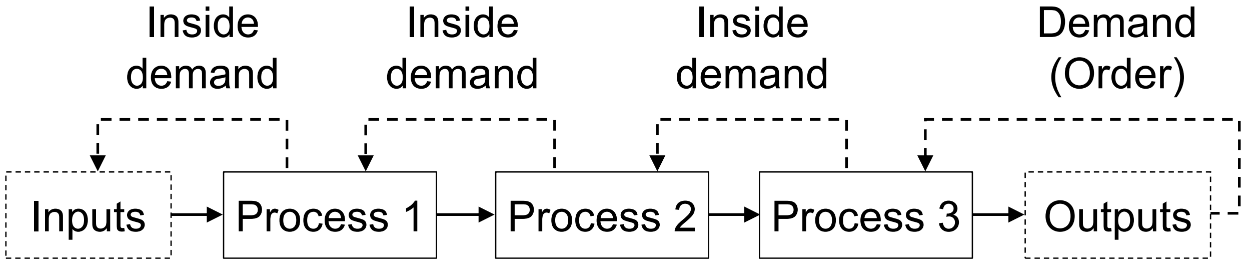
- Management of production systems



- **Management strategy**
  - PUSH



  - PULL



Basic terminology of production systems

ŠKODA AUTO University

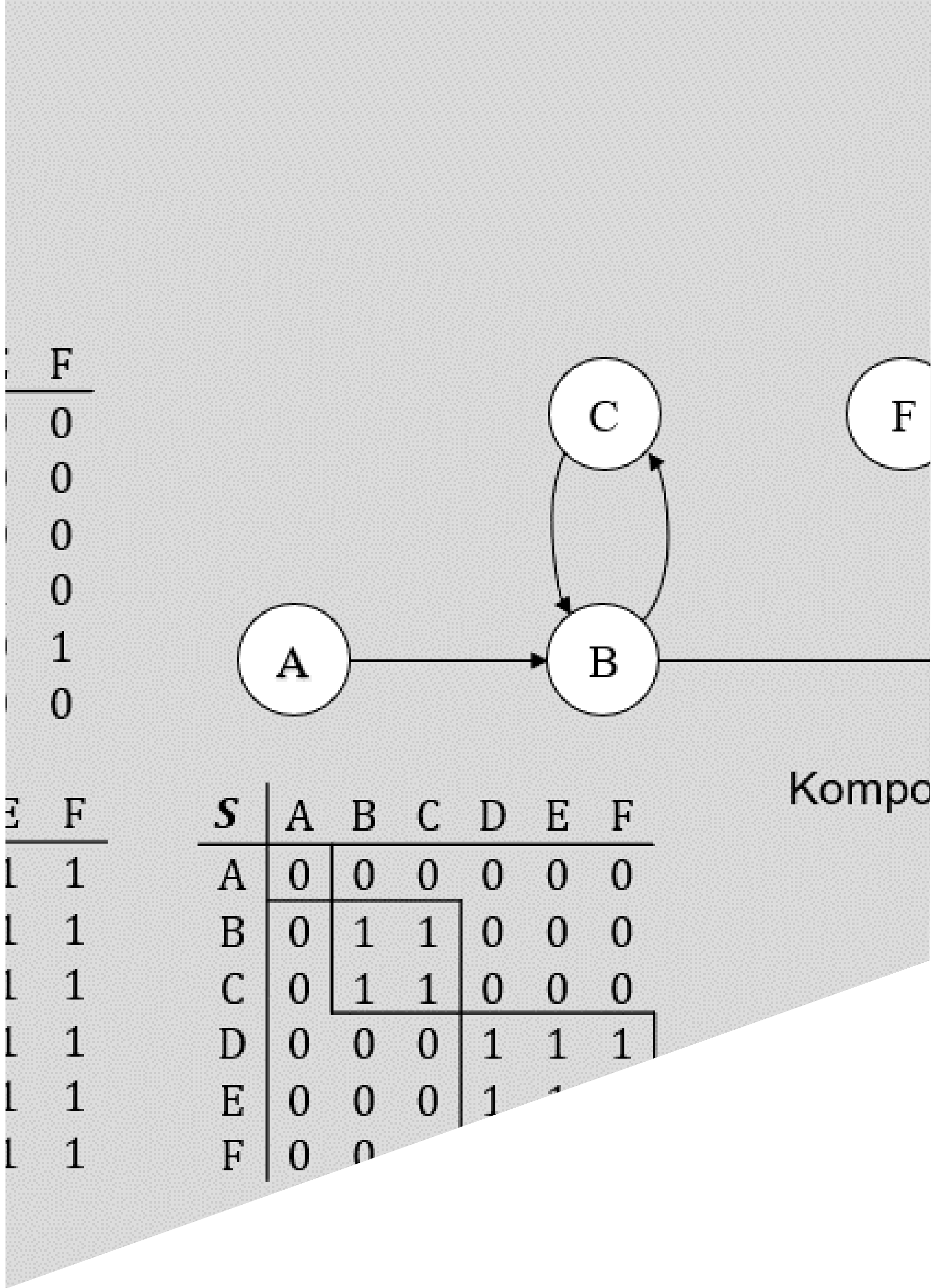
Phases of production systems analysis

- Measurement of production systems performance
  - **Indicators**
    - Economy – resource costs.
    - Efficiency – productivity (ratio of outputs to inputs), efficiency of resource involvement.
    - Effectiveness – quality of outputs:
      - Financial indicators – costs arising from poor quality or its prevention.
      - Operational indicators – percentage of rejects, size of waste, delivery delays.
      - Customer indicators – customer satisfaction.
- Improving the performance of production systems
  - **Change of production system**
    - Reengineering – step (jump) change of performance.
    - Kaizen – continuous improvement of performance – method PDCA (Plan, Do, Check, Act), Deming cycle.

ŠKODA AUTO University

2

Models of designing production systems



Models of designing production systems

ŠKODA AUTO University

Product designing

- Objectives of company
  - to bring new products to market as quickly as possible,
  - increase the level of customer satisfaction, improve quality (TQM)
  - reduce costs.
- Means for designing
  - using standards,
  - Quality Function Deployment (QFD),
  - collaborative designing.

Models of designing production systems

ŠKODA AUTO University

Product designing

- Using standards
  - VDA (Verband der Automobilindustrie),
  - modular designing – grouping of components into modules (easy and quick replacement),
  - CAD (Computer-Aided Design), CAM (Computer-Aided Manufacturing), CAE (Computer-Aided Engineering).
- Quality Function Deployment (QFD)
  - a method developed in Japan in the 1960s,
  - the company develops and produces only what the customer expects,
  - using a method known as the "house of quality",
  - the advantages of
    - deepening the orientation on customer,
    - shortening the time of product development, reducing costs of development and production,
    - motivating collaborators to work together,
    - simplicity and clarity of the tool.

Models of designing production systems

Product designing

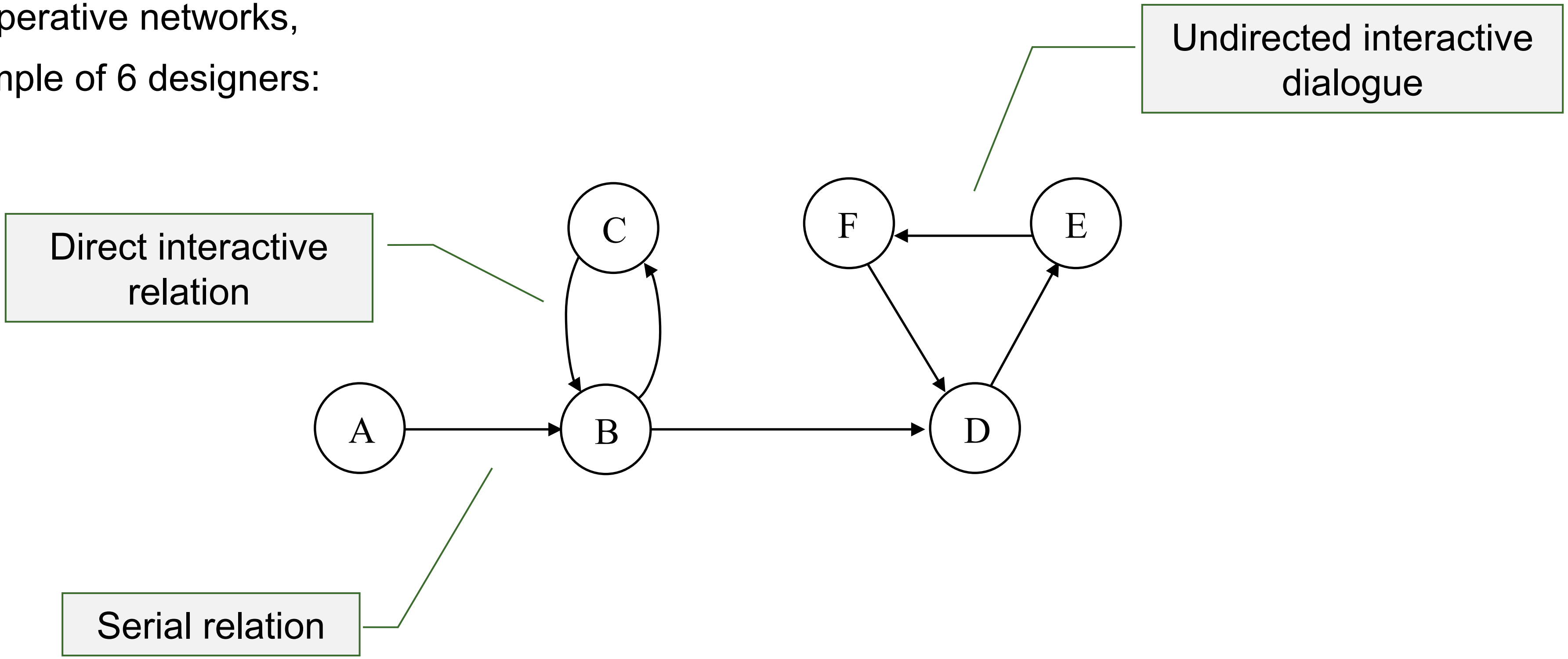
- Collaborative designing
  - several designers are involved in the design process.
  - structured process,
  - a project with co-operative activities that can take place in the following relations:
    - serial – the following in succession,
    - parallel – can be performed simultaneously,
    - interconnected – must be performed simultaneously,
  - design structure matrix $\boldsymbol{P}$:

$$p_{ij} = \begin{cases} 1 & \text{if there is a direct information flow from } \\ & i-\text{th designer to } j-\text{th designer,} \\ 0 & \text{otherwise,} \end{cases} \qquad \begin{array}{l} i = 1,2,\dots,n, \\ j = 1,2,\dots,n. \end{array}$$

Models of designing production systems

Product designing

- Collaborative designing
  - co-operative networks,
  - example of 6 designers:



Models of designing production systems

ŠKODA AUTO University

Product designing

- Analysis of information flows
  - the objective is to identify co-operating subgroups of designers,
  - boolean algebra can be used :

a	b	$a \oplus b$	$a \otimes b$
1	1	1	1
1	0	1	0
0	1	1	0
0	0	0	0

Models of designing production systems

ŠKODA AUTO University

Product designing

- Analysis of information flows
  - Relation between elements through m direct relations: :

$$p_{ij}^{(m)} = \begin{cases} 1 & \text{if there is a relation between elements } i \text{ and } j \\ & \text{as the sequence of } m \text{ direct relations,} \\ 0 & \text{otherwise,} \end{cases} \qquad \begin{aligned} i &= 1,2,\dots,n, \\ j &= 1,2,\dots,n, \\ m &= 2,3,\dots,n-1. \end{aligned}$$

  - Relation matrix:

$$\boldsymbol{R} = \boldsymbol{P} \oplus \boldsymbol{P}^{(2)} \oplus \boldsymbol{P}^{(3)} \oplus \dots \oplus \boldsymbol{P}^{(n-1)}.$$

$$r_{ij} = \begin{cases} 1 & \text{if tehere is a relation between elements } i \text{ and } j, \\ 0 & \text{oherwise,} \end{cases} \qquad \begin{aligned} i &= 1,2,\dots,n, \\ j &= 1,2,\dots,n. \end{aligned}$$

  - Two types of subsets of elements:
    - there is a mutual relation between elements,
    - no mutual relation between elements exists.

Models of designing production systems

Product designing

- Analysis of information flows
  - Strong relation matrix $\boldsymbol{S}$:

$$s_{ij} = \begin{cases} 1 & \text{if } r_{ij} = r_{ji} = 1, \\ 0 & \text{otherwise,} \end{cases} \qquad \begin{aligned} i &= 1,2,\dots,n, \\ j &= 1,2,\dots,n. \end{aligned}$$

    - The matrix is symmetric and transitive.
    - The matrix can be rearranged into a matrix with a block-diagonal structure.
    - Designers in the block interact to each other.

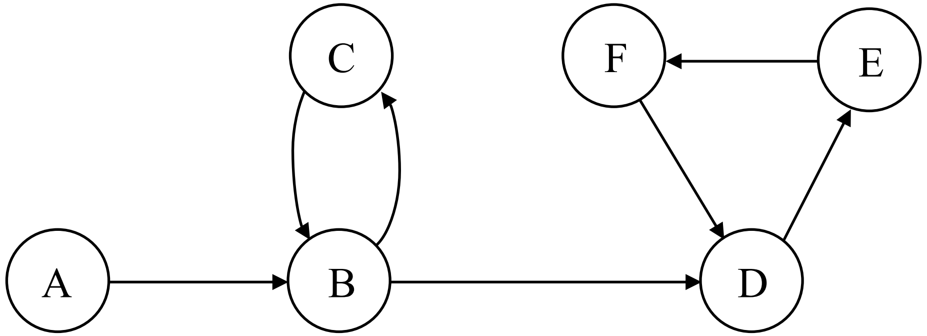
Models of designing production systems

ŠKODA AUTO University

Product designing

- Analysis of information flows
    - Example

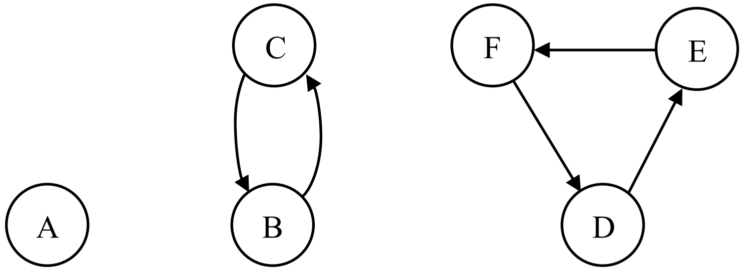
P	A	B	C	D	E	F
A	0	1	0	0	0	0
B	0	0	1	1	0	0
C	0	1	0	0	0	0
D	0	0	0	0	1	0
E	0	0	0	0	0	1
F	0	0	0	1	0	0



R	A	B	C	D	E	F
A	0	1	1	1	1	1
B	0	1	1	1	1	1
C	0	1	1	1	1	1
D	0	0	0	1	1	1
E	0	0	0	1	1	1
F	0	0	0	1	1	1

S	A	B	C	D	E	F
A	0	0	0	0	0	0
B	0	1	1	0	0	0
C	0	1	1	0	0	0
D	0	0	0	1	1	1
E	0	0	0	1	1	1
F	0	0	0	1	1	1

Components of a strong connection



Models of designing production systems

ŠKODA AUTO University

Placement of facilities

- Distance of the facility from
  - sources,
  - customers,
  - competitors,
  - other parts of system.
- Other factors
  - market – demand and competition,
  - material – transport, labour, energy, etc.,
  - non-material – ecology, social attitudes, customer habits, etc.
- Objectives
  - Connection of the new equipment with the existing production system?
  - How many types of products will the new facility produce?
  - How many new facilities will be placed?

Models of designing production systems

ŠKODA AUTO University

Placement of facilities

- Methods for selection of location
    - Break-even point analysis.
    - Multiple criteria evaluation of alternatives.
    - Method of gravity center.
    - Facility location problem – modified transportation problem.
    - Facilities location problem – quadratic assignment problem.

Models of designing production systems

ŠKODA AUTO University

Placement of facilities

- Break-even point analysis
  - Equalization of total costs and total revenues (zero profit):

$$TC = TR.$$

  - The total revenue is identical for all locations, i.e. the selection depends only on the total cost:

$$TC_i = FC_i + v_i q \qquad i = 1,2,\ldots,n.$$

  - Selection of the location for given total production size q^*:

$$TC_k = \min_{i=1,2,\ldots,n} \left(FC_i + v_i q^*\right).$$

Models of designing production systems

ŠKODA AUTO University

Placement of facilities

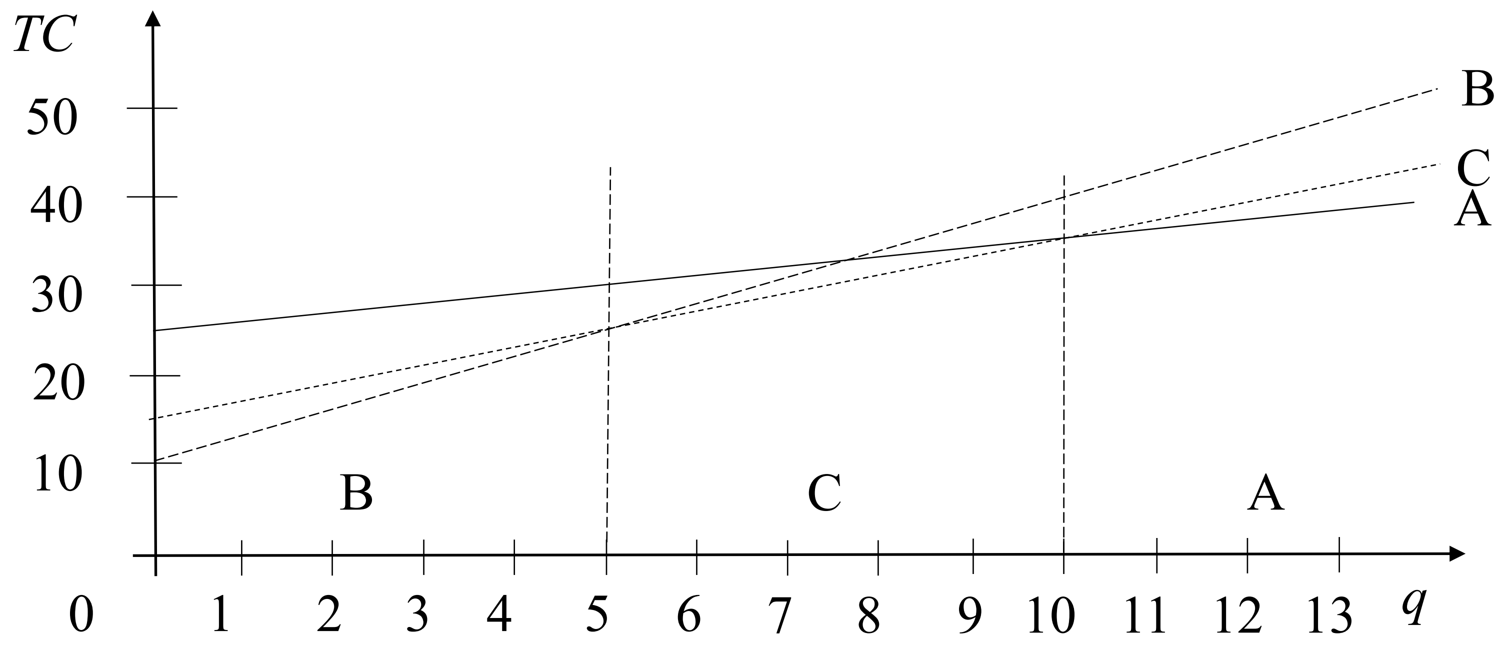
- Break-even point analysis
  - Example
    - Three potential locations for the new facility are given: A, B and C. The table shows the fixed costs in CZK per month and the variable costs in CZK per unit of production for each location.
    - Select a location for a known production of 800 units.
    - How does the selection of location depend on the production size in general?

Location	FC	v
A	25000	10
B	10000	30
C	15000	20

Models of designing production systems

Placement of facilities

- Break-even point analysis
  - Example – continued
    - Total cost are shown in 1000 CZK, production size in 100 units.



Models of designing production systems

ŠKODA AUTO University

Placement of facilities

- Multiple criteria evaluation of alternatives
  - Locations are evaluated in terms of several criteria.
  - Weighted Sum Approach (WSA):
    - Evaluation of i–th alternative according to j–th criterion:

$$y_{ij} \text{ for } i = 1,2,\ldots,n; j = 1,2,\ldots,k.$$

    - Weight of j–th criterion:

$$v_j > 0 \text{ for } j = 1,2,\ldots,k,$$

$$\sum_{j=1}^{k} v_j = 1 \text{ (normalized weights).}$$

    - Weighted sum:

$$u_i = \sum_{j=1}^{k} v_j y_{ij} \text{ for } i = 1,2,\ldots,n.$$

Models of designing production systems

ŠKODA AUTO University

Placement of facilities

- Multiple criteria evaluation of alternatives
  - Example
    - Three potential locations for the new facility are given: A, B and C. Each option is evaluated according to seven criteria. The table shows the scores for each option according to the criteria and the weight of the criteria. The company is to select the location with the highest multi-criteria score.

Location	Material	Wages	Water	Transport	Climate	Taxes	u_i
A	100	80	60	50	40	60	60
B	60	80	90	60	70	40	72
C	50	70	80	80	50	70	69
Weights	0,10	0,05	0,40	0,10	0,20	0,15	

Models of designing production systems

ŠKODA AUTO University

Placement of facilities

- Method of gravity center
  - Knowledge of existing network of parts of the system, i.e. coordinates of their locations: x_i a y_i for $i = 1,2,\ldots,n$.
  - Placement of the new facility in the center of gravity of the grid with coordinates:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \;.$$

  - Knowledge of transport volumes between facilities and the new facility: q_i for $i = 1,2,\ldots,n$.
  - Weights:

$$v_i = \frac{q_i}{Q} \qquad i = 1,2,\ldots,n, \qquad Q = \sum_{i=1}^{n} q_i \;.$$

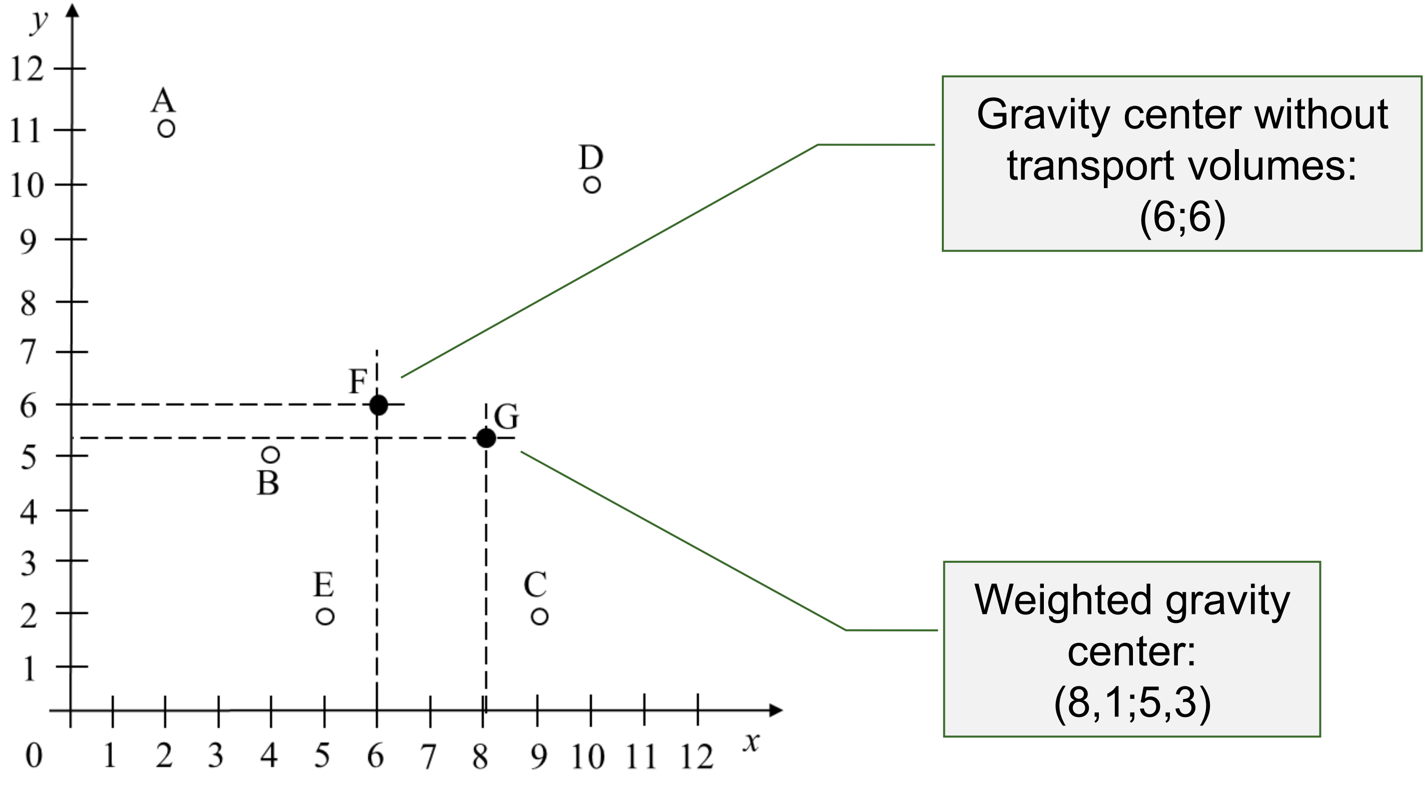
  - Coordinates of gravity center:

$$\bar{x} = \sum_{i=1}^{n} v_i x_i \qquad \bar{y} = \sum_{i=1}^{n} v_i y_i \;.$$

Models of designing production systems

Placement of facilities

- Method of gravity center
  - Example – continued



Models of designing production systems

ŠKODA AUTO University

Placement of facilities

- Facility location problem – modified transportation problem
  - Set of m potential locations for new facilities:

    a_i capacity of the facility in location i,
    f_i fixed cost for the use of location i, $i = 1,2, \dots, m$.

  - Set of n destinations (customers or internal customers):

    b_j demand of destination j, $j = 1,2, \dots, n$.

  - Transportation costs:

    c_{ij} shipping cost for transport a unit from source i to destination j

  - Variables:

$$x_i = \begin{cases} 1 & \text{if the facility is placed in location } i, \\ 0 & \text{otherwise,} \end{cases} \quad i = 1,2, \dots, m.$$

y_{ij} amount of product transported from the facility placed in location i to destination j, $i = 1,2, \dots, m$, $j = 1,2, \dots, n$.

Models of designing production systems

ŠKODA AUTO University

Placement of facilities

- Facility location problem – modified transportation problem
  - Mathematical model:

$$z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}y_{ij} + \sum_{i=1}^{m} f_i x_i \quad \rightarrow \quad \min,$$

$$\sum_{j=1}^{n} y_{ij} \leq a_i x_i \qquad i = 1,2,\dots,m,$$

$$\sum_{i=1}^{m} y_{ij} = b_j \qquad j = 1,2,\dots,n,$$

$$x_i \in \{0,1\} \qquad i = 1,2,\dots,m,$$

$$y_{ij} \geq 0 \qquad i = 1,2,\dots,m.$$

Models of designing production systems

ŠKODA AUTO University

Placement of facilities

- Facility location problem – modified transportation problem
  - Example
    - The company can use seven potential warehouses from which to transport material to its five production sites. The table gives the monthly requirements of the production sites and the monthly capacities of the warehouses (in thousands of tons). If a warehouse is used, the company has to pay the amounts indicated for its monthly rent (in thousands of euros). In addition, unit shipping costs (in euro per ton) are charged for each transport from warehouse to production site. Decide which warehouses to lease and how much material to transport between the leased warehouses and the manufacturing plants so that the total monthly cost is minimal.

Location	V1	V2	V3	V4	V5	a_i	f_i
S1	10	15	20	12	8	20	10
S2	7	10	15	22	13	25	12
S3	20	13	10	11	9	15	8
S4	15	12	21	18	16	18	9
S5	11	22	12	10	15	22	11
S6	9	13	11	18	22	30	13
S7	18	10	15	7	9	23	11
b_j	25	22	17	22	15		

Models of designing production systems

ŠKODA AUTO University

Placement of facilities

- Facility location problem – quadratic assignment problem
  - Set of n facilities:

    c_{ij} material flow between facilities i and j, $i = 1,2,\dots,n,$ $j = 1,2,\dots,n.$
  - Set of n locations (workplaces):

    d_{kl} distance between locations k and l, $k = 1,2,\dots,n,$ $l = 1,2,\dots,n.$
  - Variables:

$$x_{ik} = \begin{cases} 1 & \text{if } i\text{-th facility is placed on } k\text{-th location,} \\ 0 & \text{otherwise,} \end{cases} \qquad i = 1,2,\dots,n, \quad k = 1,2,\dots,n.$$

Models of designing production systems

Placement of facilities

- Facility location problem – quadratic assignment problem
  - Mathematical model:

$$z = \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} c_{ij} d_{kl} x_{ik} x_{jl} \quad \rightarrow \quad \min,$$

$$\sum_{k=1}^{n} x_{ik} = 1 \qquad i = 1,2,\dots,n,$$

$$\sum_{i=1}^{n} x_{ik} = 1 \qquad k = 1,2,\dots,n,$$

$$x_{ik} \in \{0,1\} \qquad \begin{array}{l} i = 1,2,\dots,n, \\ k = 1,2,\dots,n. \end{array}$$

Models of designing production systems

ŠKODA AUTO University

Placement of facilities

- Facility location problem – quadratic assignment problem
  - Linearization of mathematical model:

$$y_{ijkl} = \begin{cases} 1 & \text{if } i\text{-th facility is placed on } k\text{-th location and } j\text{-th facility is placed on } l\text{-th location,} \\ 0 & \text{otherwise,} \end{cases} \qquad i,j,k,l = 1,2,\ldots,n.$$

$$y_{ijkl} \geq x_{ik} + x_{jl} - 1 \qquad i,j,k,l = 1,2,\ldots,n,$$

$$z = \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} c_{ij} d_{kl} y_{ijkl} \quad \rightarrow \quad \min.$$

Models of designing production systems

Placement of facilities

- Facility location problem – quadratic assignment problem
  - Example
    - The company intends to build 5 warehouses in 5 cities. In the first table the distances (in km) between the cities are given, in the second table the number of drives that must be made between the warehouses in one month. The aim is to decide which warehouse will be set up in which city in order to minimize the total transportation costs.

Distance	City1	City2	City3	City4	City5
City1	0	50	60	130	100
City2	50	0	70	150	120
City3	60	70	0	80	40
City4	130	150	80	0	50
City5	100	120	40	50	0

Rides	Warehouse1	Warehouse2	Warehouse3	Warehouse4	Warehouse5
Warehouse1	0	10	15	12	8
Warehouse2	9	0	18	16	10
Warehouse3	20	8	0	10	12
Warehouse4	10	15	11	0	22
Warehouse5	17	12	9	11	0

Models of designing production systems

ŠKODA AUTO University

Product schedule

- Basic terms and notation
  - Batch and mass production.
  - Each product requires the same sequence of operations.
  - The operations take place on facilities arranged in a production line.
  - All input parameters are integer values.

$\boldsymbol{A}$	set of n operations,
t_i	duration of i-th operation $(i = 1,2, \dots, n)$,
$\boldsymbol{A}_j \subset \boldsymbol{A}$	set of operations realized on j-th workplace $(j = 1,2, \dots, m)$,
c	takt time,
$i \rightarrow k$	precedence relation, i-th operations precedes k-th operation $(i, k = 1,2, \dots, n; i \neq k)$,
z	number of unused (idle) time units on the whole line,
z_j	number of unused (idle) time units on j-th workplace $(j = 1,2, \dots, m)$.

Models of designing production systems

ŠKODA AUTO University

Product schedule

- Basic terms and notation
  - Product workload

$$t = \sum_{i=1}^{n} t_i \,.$$

  - Sum of durations of opertions on j-th workplace

$$t(\boldsymbol{A}_j) = \sum_{i \in \boldsymbol{A}_j} t_i \qquad j = 1,2, \ldots, m.$$

Models of designing production systems

ŠKODA AUTO University

Product schedule

- Feasible schedule
  - Example
    - On a production line with 3 workplaces, where the product is produced, 8 operations with the following operation times (in min) have to be scheduled: 2,1,7,3,6,5,4,2. The 7th operation in the sequence with the duration of 4 min has to be scheduled after the operation with the duration of 1 min, i.e. the precedence relation 2 → 7 is valid. Takt time was set to 10 min.

a) What is the product worlkoad?

b) Decide, whether the following schedule is feasible:

$$\boldsymbol{A}_1 = \{3, 4\},$$

$$\boldsymbol{A}_2 = \{5, 7\},$$

$$\boldsymbol{A}_3 = \{1, 2, 6, 8\}.$$

c) Determine a number of idle minutes on the production line.

d) What is the production line efficiency?

Models of designing production systems

ŠKODA AUTO University

Product schedule

- Feasible schedule
  - Conditions for a feasible schedule
    - All operations must be scheduled

$$\bigcup_{j=1}^{m} \boldsymbol{A}_j = \boldsymbol{A},$$

    - Each operation must be assigned to exactly one workplace

$$\boldsymbol{A}_j \bigcap \boldsymbol{A}_k = \emptyset \qquad j,k = 1,2,\dots,m, j \neq k,$$

    - All operation assigned to a workplace must be done in takt time

$$t(\boldsymbol{A}_j) \leq c \qquad j = 1,2,\dots,m.$$

    - All precedence constraints must be satisfied

$$i \to h, i \in \boldsymbol{A}_j \wedge h \in \boldsymbol{A}_k \Rightarrow j \leq k \qquad \begin{matrix} i,h = 1,2,\dots,n, \\ j,k = 1,2,\dots,m. \end{matrix}$$

Models of designing production systems

ŠKODA AUTO University

Product schedule

- Optimal schedule
  - Criteria
    - Minimization of a number of workplaces for given value of takt time

      $$m \to \min \qquad \text{pro } c = \text{const.}$$

    - Minimization of takt time for given number of workplaces

      $$c \to \min \qquad \text{pro } m = \text{const.}$$

    - Minimization of idle time on the whole production line

      $$z = \sum_{j=1}^{m} z_j = \sum_{j=1}^{m} \left(c - t(\boldsymbol{A}_j)\right) = mc - \sum_{j=1}^{m} t(\boldsymbol{A}_j) = mc - t \quad \to \min.$$

    - Maximization of the production line efficiency

      $$E = \frac{t}{mc} 100\ \% \ \to \ \max.$$

Models of designing production systems

ŠKODA AUTO University

Product schedule

- Minimization of a number of workplaces
  - Generally: $1 \leq m \leq n$.
  - For $m = 1$:

$$c = t$$
$$z = z_1 = 0$$
$$E = 100\ \%$$

  - For $m = n$:

$$c = t_{\max} = \max_{i=1,2,\ldots,n} t_i$$

$\lceil x \rceil$ ceiling

$\lfloor x \rfloor$ floor

  - If a takt time is given $c =$ const., then

$$1 \leq m_{\min} \leq m \leq m_{\max} \leq n \qquad m_{\min} = \left\lceil \frac{t}{c} \right\rceil \quad m_{\max} = \left\lfloor \frac{t}{t_{\max}} \right\rfloor$$

Models of designing production systems

Product schedule

- Minimization of a number of workplaces
  - Example
    - The company wants to schedule a product on the line, the production consists of 6 operations with durations (in min): 2,4,6,3,5,3. There are no technological interdependencies between the operations, i.e. no precedence relation is defined. The takt time is set to 8 min.
      a) What is a product workload?
      b) Determine minimal and maximal number of workplaces on production line and find any feasible schedule.
      c) For found schedule, determine a number of idle minutes on the whole production line and calculate its efficiency.

Models of designing production systems

ŠKODA AUTO University

Product schedule

- Minimization of takt time
  - Generally: $1 \leq c \leq t.$
  - For $m = 1$:

$$c = t$$
$$E = 100\ \%$$

  - For $m = n$:

$$\text{if } t_i = 1 \text{ for all } i = 1,2,\dots,n \quad \Rightarrow \quad c = 1, \quad E = 100\ \%$$

  - For $1 < m < n$:
    - Theoretically achievable minimum takt time: $c_{\min} = \max\left(\left\lceil \frac{t}{m} \right\rceil, t_{\max}\right)$

Models of designing production systems

ŠKODA AUTO University

Product schedule

- Minimization of takt time
  - Highest possible takt time at which required production size Q can be achieved in working period of legth T:

$$c_{\max} = \left\lfloor \frac{T}{Q} \right\rfloor.$$

  - Realized takt time:

$$1 \leq c_{\min} \leq \mathrm{c} \leq c_{\max} \leq t.$$

Models of designing production systems

Product schedule

- Minimization of takt time
  - Example
    - The company wants to schedule a product with 5 operations and their durations (in min): 5,1,3,4,2 on 3 workplaces. There is no precedence relation between the operations.
      a) What is the theoretically achievable minimum takt time?
      b) Find a feasible schedule for this value.
      c) What is the production line efficiency?
      d) How do the takt time and the production line efficiency change for durations: 2,2,2,5,4?

Models of designing production systems

Product schedule

- Minimization of takt time
  - Example
    - The company plans to achieve the daily production size Q=100 units in the working period of length T=9 hours. Production of the product consists of 7 operations with durations (in min): 2,4,5,4,3,1,2.
      a) What is the highest possible takt time at which the desired daily production can still be achieved?
      b) Is it possible to find a feasible schedule for this takt time with the existing 5 workplaces?
      c) What will be the efficiency of the whole production line?

Models of designing production systems

ŠKODA AUTO University

Product schedule

- Heuristic methods
  - Problem definition
    - Production is divided into n operations.
    - The sequence of operations is given by precedence relations. For their graphical representation, a directed graph can be used.
    - For each operation i its duration t_i is given.
    - Total daily production size Q is required.
    - Length of working period is T.

Models of designing production systems

ŠKODA AUTO University

Product schedule

- Heuristic methods
  - General procedure
    - Step 1
      - Calculate maximal takt time: $c_{\max} = \left\lfloor \frac{T}{Q} \right\rfloor$.
    - Step 2
      - Calculate product workload and theoretical minimal number of workplaces:

$$t = \sum_{i=1}^{n} t_i, \qquad m_{\min} = \left\lceil \frac{t}{c_{\max}} \right\rceil.$$

    - Step 3
      - Use one of 4 rules to assign operations to individual workplaces (see below).
    - Step 4
      - Calculate the production line efficiency: $E = \frac{t}{mc} 100\ \%$.

Models of designing production systems

ŠKODA AUTO University

Product schedule

- Heuristic methods
  - Rules for the assignment of operations to workplaces (see step 3):
    - longest duration,
    - maximal number of predecessors,
    - maximal number of successors,
    - maximal weight:

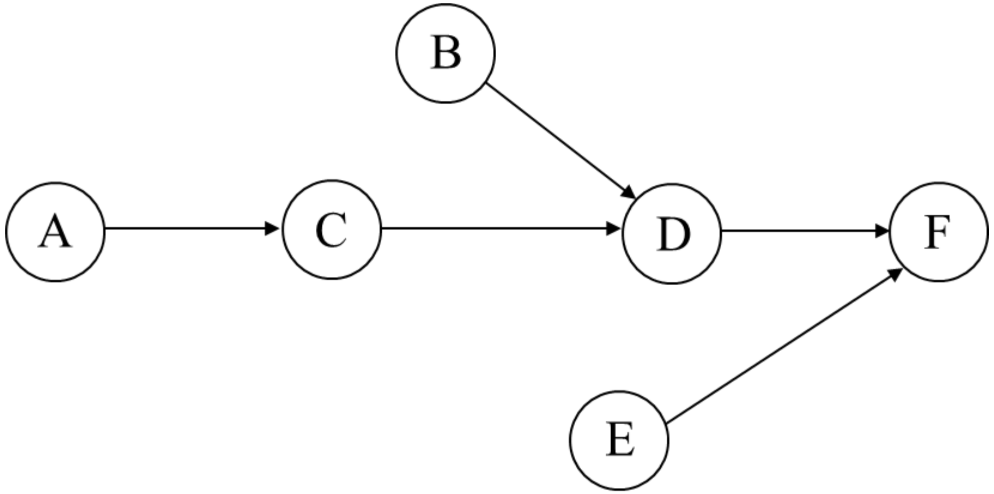
$$w_i = t_i + \sum_{j \in \boldsymbol{N}_i} w_j \qquad i = 1,2,\dots,n,$$

where N_i is a set of all immediate successors of i-th operation.

Models of designing production systems

Product schedule

- Heuristic methods
  - Example
    - The company plans to produce 12 units of product per day, which production consists of 6 operations. All precedence relations are illustrated by the directed graph in the figure.



Models of designing production systems

ŠKODA AUTO University

Product schedule

- Heuristic methods
  - Example – continued
    - The table specifies the duration of each operation. In addition, the sets of the immediately preceding and immediately following operations are obtained from the graph. For further calculation, the numbers of all preceding and all following operations are additionally determined in the table for each activity. The length of the daily working period is 6 hours.

Operation	t_i	Predecessors	Successors	Number of predecessors	Number of successors
A	12	-	C	0	3
B	15	-	D	0	2
C	8	A	D	1	2
D	10	B, C	F	3	1
E	20	-	F	0	1
F	14	D, E	-	5	0

Models of designing production systems

ŠKODA AUTO University

Product schedule

- Mathematical model

$$x_{ij} = \begin{cases} 1 & \text{if } i\text{-th operation is assigned to } j\text{-th workplace,} \\ 0 & \text{otherwise,} \end{cases} \qquad \begin{array}{l} i = 1,2,\dots,n, \\ j = 1,2,\dots,m. \end{array}$$

$$z = \sum_{j=1}^{m} z_j = \sum_{j=1}^{m}\left(c - \sum_{i=1}^{n} t_i x_{ij}\right) = mc - t \rightarrow \ \text{min},$$

$$\sum_{i=1}^{n} t_i x_{ij} \le c \qquad j = 1,2,\dots,m,$$

$$\sum_{j=1}^{m} x_{ij} = 1 \qquad i = 1,2,\dots,n,$$

$$x_{kh} \le \sum_{j=1}^{h} x_{ij} \qquad h = 1,2,\dots,m, i \rightarrow k,$$

3

Models of management of production systems

$$z = \sum_{j=1}^{n} d_j^+, \quad \rightarrow \min,$$

$$C_j \leq F_{\max} \qquad j = 1,2,\ldots,n,$$

$$C_i + t_j \leq C_j + M(1 - x_{ij}) \qquad i,j =$$

$$x_{ij} + x_{ji} = 1 \qquad i,j = 1,2,\ldots,n, i$$

$$C_j - d_j^+ + d_j^- = d_j \qquad j = 1,2,\ldots,$$

$$C_j \geq t_j \qquad j = 1,2,\ldots,n,$$

$$d_j^+, d_j^- \geq 0 \qquad j = 1,2,\ldots,n,$$

$$x_{ij} \in \{0,1\} \qquad i,j =$$

Models of management of production systems

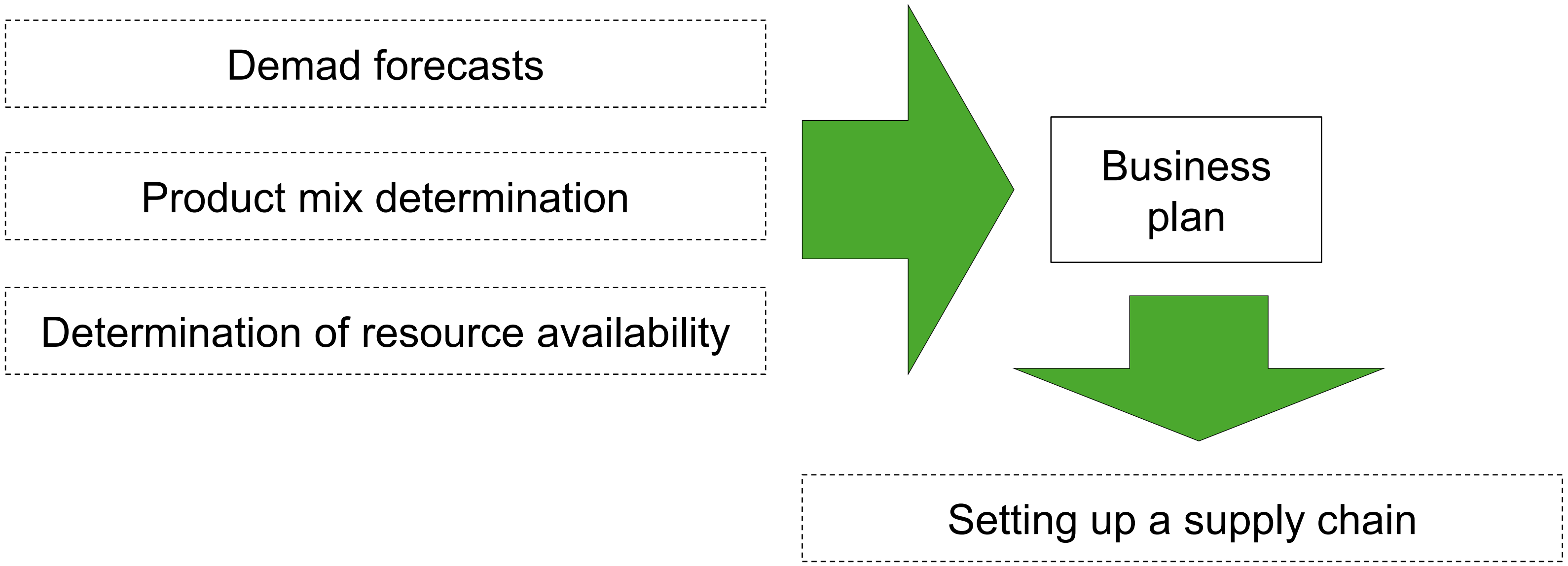
Management of production systems

- Steps of management od production systems
  - Production planning – determination of production capacity in the long term.
  - Production scheduling – the schedulling of production to individual facilities over a shorter time horizon.
  - Production control – implementation of schedules, done in real time.

Models of management of production systems

Production planning

- Planning of aggregated production
  - Satisfaction of demand.
  - Demand size is determined in aggregated units → production size is also determined in aggregated units.
  - Assessment of production capacities and their possible increase.



Models of management of production systems

ŠKODA AUTO University

Production planning

- Planning of aggregated production
  - Decision-making problem

D_t	size of aggregated demand in period t,
P_t	production size in period t,
W_t	level of human resources (workforce) in period t,
I_t	inventory level in period t.

(P_t, W_t, I_t: Control variables)

  - Strategies for setting of control variables:
    - **Constant level**. We do not change W_t and, therefore, neither P_t. There may be overproduction (higher storage costs) or underproduction (lost profits) relative to demand.
    - **Tracking strategy**. Responding to the size of demand by changing workforce level W_t so that production size P_t matches the change in the size of demand as closely as possible. Higher costs associated with firing and hiring workers.
    - **Combined strategy**. Partial tracking of demand size to use both approaches.

Models of management of production systems

ŠKODA AUTO University

Production planning

- Planning of aggregated production
  - Mathematical model

N	number of groups into which the products are sorted ($i = 1,2, \dots , N$),
T	number of periods into which the time horizon is divided ($t = 1,2, \dots , T$),
M	number of source types ($m = 1,2, \dots , M$),
D_{it}	demand forecast for products group i in periods t,
R_{mt}	capacity of the source of type m in period t,
r_{im}	number of units of the source type m, required by the products group i,
I_{it}	inventory level of products group i at the end of period t,
I_{i0}	initial inventory level of products group i,
I_{it}^{*}	safety stock of products group i at the end of period t,
c_i	unit holding cost for products group i per time period,
x_{it}	production size for products group i in period t.

Models of management of production systems

ŠKODA AUTO University

Production planning

- Planning of aggregated production
  - Mathematical model – continued

$$z = \sum_{t=1}^{T} \sum_{i=1}^{N} c_i \, I_{it} \ \rightarrow \min,$$

$$I_{i,t-1} + x_{it} - I_{it} = D_{it} \qquad \begin{matrix} i = 1,2, \dots, N, \\ t = 1,2, \dots, T, \end{matrix}$$

$$\sum_{i=1}^{N} r_{im} x_{it} \le R_{mt} \qquad \begin{matrix} m = 1,2, \dots, M, \\ t = 1,2, \dots, T, \end{matrix}$$

$$I_{it} \ge I_{it}^{*} \qquad \begin{matrix} i = 1,2, \dots, N, \\ t = 1,2, \dots, T, \end{matrix}$$

$$x_{it} \ge 0 \qquad \begin{matrix} i = 1,2, \dots, N, \\ t = 1,2, \dots, T. \end{matrix}$$

Models of management of production systems

ŠKODA AUTO University

Production planning

- Planning of aggregated production
  - Mathematical model – extension for overloading sources

s_{mt} size of the overload of the source type m in period t,

p penalty for the overload of the source,

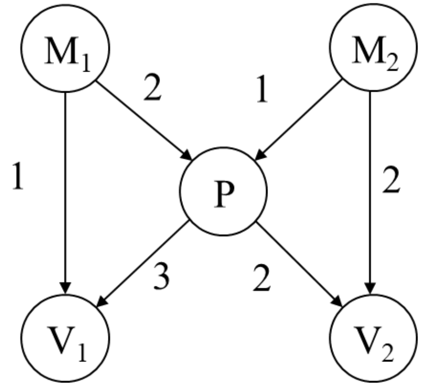
$$\sum_{i=1}^{N} r_{im} x_{it} \leq R_{mt} + s_{mt} \qquad \begin{matrix} m = 1,2,\dots,M, \\ t = 1,2,\dots,T, \end{matrix}$$

$$z = \sum_{t=1}^{T} \left(\sum_{i=1}^{N} c_i \, I_{it} + p \sum_{m=1}^{M} s_{mt} \right) \to \text{min.}$$

Models of management of production systems

Production planning

- Multi-level planning
  - Techniques
    - Input-output analysis
    - Material Requirement Planning (MRP).
    - Manufacturing Resource Planning (MRP II).
  - Gozinto graph
    - Nodes
      - Inputs: material, semi-finished products.
      - Outputs: semi-finished products, final products.
    - Arcs (directed)
      - Transformation of material and products.
      - Arcs evaluation: number of the input units transformed to the output unit.



Models of management of production systems

ŠKODA AUTO University

Production planning

- Multi-level planning
  - **Input-output analysis**

    $\boldsymbol{A}$ input-output matrix of coefficients a_{ij} (amount of i-th input item necessary to produce a unit of j-th output item),

    $\boldsymbol{b}$ vector of required size of final production,

    $\boldsymbol{y}$ vector of all production input and output values,

    - Leontief model: $\boldsymbol{y} = (\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{b}$.
    - Optimization model:

$$z = f(\boldsymbol{y}) \rightarrow \min\ (\max),$$

$$(\boldsymbol{I} - \boldsymbol{A})\boldsymbol{y} = \boldsymbol{b},$$

$$\boldsymbol{y} \geq \boldsymbol{0}.$$

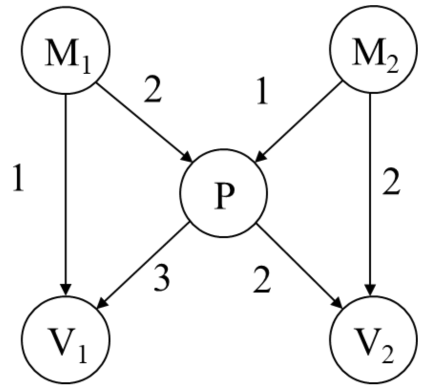
Models of management of production systems

Production planning

- Multi-level planning
  - **Input-output analysis**
    - Example
      - The company produces two final products V1 and V2 in 20 and 30 units. Two types of materials M1 and M2 and semi-finished product P are used. Product structure is expressed by Gozinto graph. Calculate all production input and output values:

        a) Use the Leontief model.

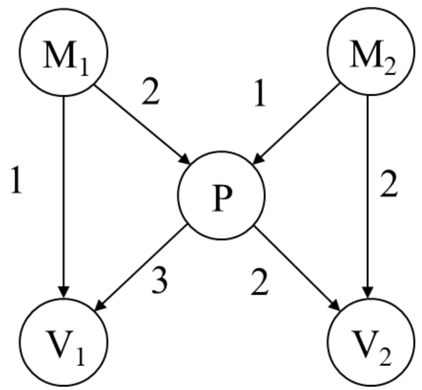
        b) Use the optimization model.



Models of management of production systems

Production planning

- Multi-level planning
  - **Input-output analysis**
    - Example – continued



$\boldsymbol{A}$	M_1	M_2	P	V_1	V_2
M_1	0	0	2	1	0
M_2	0	0	1	0	2
P	0	0	0	3	2
V_1	0	0	0	0	0
V_2	0	0	0	0	0

	$\boldsymbol{b}$
M_1	0
M_2	0
P	0
V_1	20
V_2	30

$(\boldsymbol{I}-\boldsymbol{A})^{-1}$	M_1	M_2	P	V_1	V_2
M_1	1	0	2	7	4
M_2	0	1	1	3	4
P	0	0	1	3	2
V_1	0	0	0	1	0
V_2	0	0	0	0	1

	$\boldsymbol{y}$
M_1	260
M_2	180
P	120
V_1	20
V_2	30

Models of management of production systems

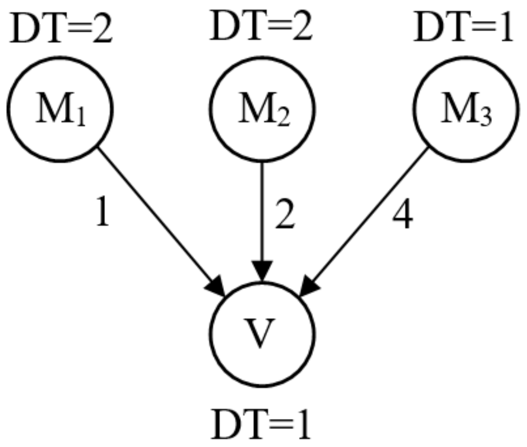
Production planning

- Multi-level planning
  - **MRP**
    - **MPS – Master Production Schedule**
      - Transformation of aggregated plan into detailed production plan.
      - What will be produced, when and in what sizes (special products and capacity requirements during the considered time period are determined).
    - **BOM – Bill of Materials**
      - List of all materials, components and semi-finished products.
      - Assembly tree of product structure (Gozinto graph).
      - Inventory levels (stock), delivery times (lead times), orders, safety stock levels.

Models of management of production systems

Production planning

- Multi-level planning
  - **MRP**
    - Example
      - For 1 unit of product V, 1 unit of material M1, 2 units of material M2 and 4 units of material M3 are required. The delivery times (DT, in weeks) for each material type and for product V (production time) are shown in the figure (Gozinto graph). Product V can start production when all the material is available. The company has to deliver 80 units of product V in week 4 and 160 units of product V in week 7. The initial inventory level is 50 units of product V, 50 units of material M1, 40 units of material M2 and 140 units of material M3.
        The objective is to plan the material requirements for the whole period so that the demand for product V is met.



Models of management of production systems

ŠKODA AUTO University

Production planning

- Multi-level planning
  - **MRP**
    - Example – continued

Delivery of product V	Week	1	2	3	4	5	6	7
	Number of products				80			160

Product V DT=1		1	2	3	4	5	6	7
	Total requirement				80			160
	Stock	50	50	50	50			
	Net requirement				30			160
	Order			30			160	

Models of management of production systems

ŠKODA AUTO University

Production planning

- Multi-level planning
  - **MRP**
    - Example – continued

	Week	1	2	3	4	5	6	7
Material M_1 DT=2	Total requirement			30			160	
	Stock	50	50	50	20	20	20	
	Net requirement						140	
	Order				140			
Material M_2 DT=2	Total requirement			60			320	
	Stock	40	40	40				
	Net requirement			20			320	
	Order	20			320			
Material M_3 DT=1	Total requirement			120			640	
	Stock	140	140	140	20	20	20	
	Net requirement						620	
	Order					620		

Models of management of production systems

ŠKODA AUTO University

Production scheduling

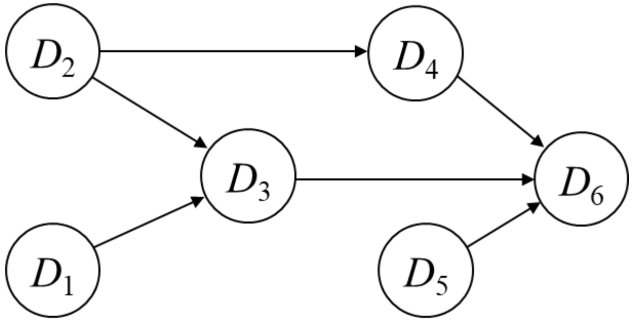
- Basic terms
  - Elements
    - machines (processors): $P_1, P_2, \dots, P_m$ or $1,2,\dots,m,$
    - jobs: $D_1, D_2, \dots, D_n$ or $1,2,\dots,n,$
  - Model classification
    - Number of machines
      - single machine models,
      - multiple machine models, machines are arranged in parallel or in series.
    - Random consideration:
      - deterministic models,
      - stochastic (probabilistic) models.
    - Knowledge of all jobs:
      - static,
      - dynamic.

Models of management of production systems

ŠKODA AUTO University

Production scheduling

- Basic terms
  - Precedence constraints (relations)
    - Expression of relationship between jobs during processing.
    - $D_j \rightarrow D_k$ means that processing of job D_k can start after processing D_j is completed.
    - Precedence graph – graphical representation of all precedence relations in the problem.



Models of management of production systems

Production scheduling

- Basic terms
  - Preemptions
    - the division of a job into two or more time intervals with a time gap between them,
    - it is allowed to interrupt the processing of a job at any point in time and continue processing on another machine,
    - interruption of the job usually occurs after the completion of an ongoing operation.
  - Idle period
    - a time interval in which no job is processed, i.e. no operation is performed,
    - the idle period may be planned (machines maintenance), it may occur due to inefficient balancing of the production line, it may be forced (failures).
  - Schedule
    - arrangement of jobs in time,
    - Gantt chart is a graphical representation of the schedule of jobs on a time axis.

Models of management of production systems

Production scheduling

- Basic terms
  - Models of network analysis (project management)
    - a single job, called a project,
    - operations are called activities,
    - each activity runs on a separate machine,
    - the relationship of activities can be expressed by a network,
    - time and cost analysis of projects, scheduling of shared resources.

Models of management of production systems

ŠKODA AUTO University

Production scheduling

- Production scheduling models
  - Notation in models
    - Given *parameters* are denoted by lowercase characters.
    - *Variables* and *calculated values* are denoted by uppercase characters.
  - Parameters – for each job D_j it is possible to set the following data $(j = 1,2,\dots,n)$

t_j Processing time of job j. If a job consists of several operations, their durations are known as well.

r_j Release time (ready time), it is the earliest time at which job j can start its processing.

d_j Due time of job j. It is committed completion time. Completion a job after its due time is allowed, but then a penalty is incurred. When a due time must be met, it is referred to as a deadline.

w_j Weight of job j to express it importance (e.g. unit cost or penalty for delay performed).

Models of management of production systems

ŠKODA AUTO University

Production scheduling

- Production scheduling models
    - Feasible schedule
        - No machine can process multiple jobs simultaneously.
        - No job can be processed on multiple machines simultaneously.
        - No job processing can start before its release time.
        - All precedence relations must be respected for all jobs.
        - Other conditions must be respected
            - no interruption is allowed in processing,
            - jobs must be processed on a special machine,
            - due times must be respected as deadlines,

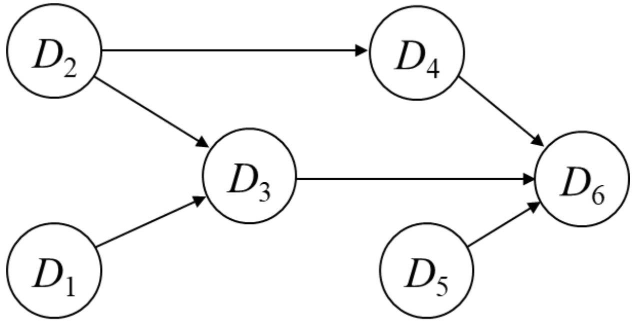
            etc.

Models of management of production systems

ŠKODA AUTO University

Production scheduling

- Production scheduling models
  - Example
    - Consider 6 jobs whose precedence graph is shown in the figure. Schedule these jobs on 2 parallel machines. The table shows the processing times of all jobs (in hours).

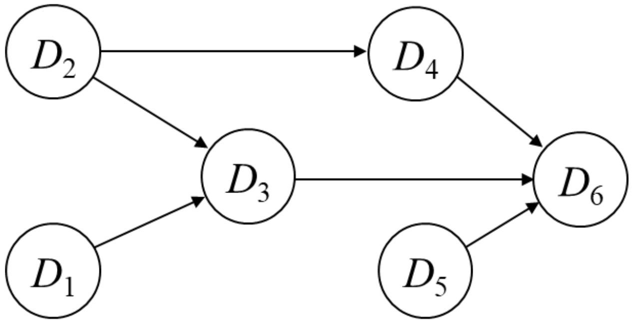


Job	t_j
D_1	3
D_2	2
D_3	1
D_4	2
D_5	2
D_6	1

Models of management of production systems

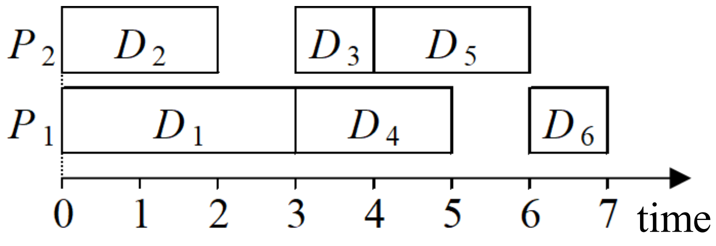
Production scheduling

- Production scheduling models
  - Example – continued

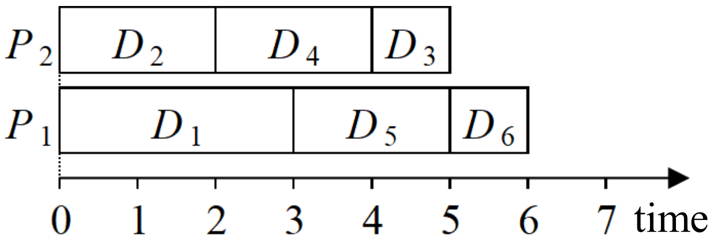


Job	t_j
D_1	3
D_2	2
D_3	1
D_4	2
D_5	2
D_6	1

- 3 idle periods



- 1 idle period



Models of management of production systems

Production scheduling

- Production scheduling models
  - Variables (calculated values)

C_j completion time of j-th job,

$F_j = C_j - r_j$ flow time determines time that j-th job spends in system,

$L_j = C_j - d_j$ lateness of job j is a difference between completion time and due time,

- for earliness it is valid: $L_j < 0$,
- for tardiness it is valid: $L_j > 0$,

$T_j = \max\left(0, L_j\right)$ tardiness of job j,

$\delta(T_j) = 1$ if $T_j > 0$,

$\delta(T_j) = 0$ if $T_j = 0$.

  - Optimal production schedule is such a feasible schedule, which minimizes selected criterion.

Models of management of production systems

ŠKODA AUTO University

Production scheduling

- Production scheduling models
  - Criteria:
    - Makespan is time all jobs will have been completed, i.e. completion time of last processed job

$$C_{\max} = \max_{j=1,2,\ldots,n} \{C_j\}, \qquad C_{\max}(w) = \max_{j=1,2,\ldots,n} \{w_j C_j\}.$$

    - Mean completion time

$$\bar{C} = \frac{1}{n}\sum_{j=1}^{n} C_j\,, \qquad \bar{C}(w) = \sum_{j=1}^{n} w_j C_j\,.$$

    - Maximal flow time (makespan)

$$F_{\max} = \max_{j=1,2,\ldots,n} \{F_j\}, \qquad F_{\max}(w) = \max_{j=1,2,\ldots,n} \{w_j F_j\}.$$

Models of management of production systems

ŠKODA AUTO University

Production scheduling

- Production scheduling models
  - Criteria:
    - Mean flow time

$$\bar{F} = \frac{1}{n}\sum_{j=1}^{n} F_j\,, \qquad \bar{F}(w) = \sum_{j=1}^{n} w_j F_j\,.$$

    - Maximum lateness

$$L_{\max} = \max_{j=1,2,\dots,n}\{L_j\}, \qquad L_{\max}(w) = \max_{j=1,2,\dots,n}\{w_j L_j\}.$$

    - Mean lateness

$$\bar{L} = \frac{1}{n}\sum_{j=1}^{n} L_j\,, \qquad \bar{L}(w) = \sum_{j=1}^{n} w_j L_j\,.$$

Models of management of production systems

ŠKODA AUTO University

Production scheduling

- Production scheduling models
  - Criteria:
    - Maximum tardiness

$$T_{\max} = \max_{j=1,2,\ldots,n} \{T_j\}, \qquad T_{\max}(w) = \max_{j=1,2,\ldots,n} \{w_j T_j\}.$$

    - Mean tardiness

$$\bar{T} = \frac{1}{n}\sum_{j=1}^{n} T_j\,, \qquad \bar{T}(w) = \sum_{j=1}^{n} w_j T_j\,.$$

    - Number of overdue jobs

$$N = \sum_{j=1}^{n} \delta(T_j), \qquad N(w) = \sum_{j=1}^{n} w_j \delta(T_j).$$

Models of management of production systems

ŠKODA AUTO University

Production scheduling

- Production scheduling models
  - Model classification $\alpha|\beta|\gamma$

α: 1 system with single machine,

P_m system with m parallel machines,

F_m flow-shop system with m serial machines, all jobs are processed in the same order (production line in mass production),

O_m open-shop system, system with m serial machines, jobs are processed in any order,

J_m job-shop system, system with m serial machines, each job is processed in fix order, but this order can differ for each job (production to order/contract).

Models of management of production systems

ŠKODA AUTO University

Production scheduling

- Production scheduling models
  - Model classification $\alpha|\beta|\gamma$

β: r_j release times of jobs do not have to be zero,

$prmp$ jobs processing can be interrupted,

$prec$ in the model precedence relations are defined,

γ: selected criterion.

Models of management of production systems

ŠKODA AUTO University

Production scheduling

- Single machine scheduling models
  - Model $1||Z$
    - There is single processor in the system.
    - There are n jobs processed, each consists of one operation.
    - Symbol Z represents generally any criterion.
    - For each job $j = 1,2,\ldots,n$ processing time t_j is given.
    - Constraints:
      - S_1: At the beginning, all n jobs are available, i.e. for each job $j = 1,2,\ldots,n$ ready time is $r_j = 0$.
      - S_2: There are no precedence relations between jobs.
      - S_3: Processing times of jobs are independent of the order of their processing.
    - No interruptions of jobs are allowed, there are no idle times in the schedule.
    - Notation "$[i] = j$" means that j-th job is processed in i-th place in order of the schedule.

Models of management of production systems

ŠKODA AUTO University

Production scheduling

- Single machine scheduling models
  - Model $1||F_{\max}$
    - All optimal schedules are equivalent.
    - Criterion value (maximal flow time) is independent of the order of jobs in schedule:

$$F_{\max} = \sum_{j=1}^{n} t_j \,.$$

    - A mathematical model for considering all due times as deadlines:

$$x_{ij} = \begin{cases} 1 & \text{if } i\text{-th job is processed at any time before } j\text{-th job,} \\ 0 & \text{oherwise,} \end{cases} \qquad \begin{matrix} i = 1,2,\dots,n, \\ j = 1,2,\dots,n. \end{matrix}$$

d_j^+ tardiness of job

d_j^- lateness of job

Models of management of production systems

ŠKODA AUTO University

Production scheduling

- Single machine scheduling models
  - Model $1||F_{\max}$ – continued

$$z = \sum_{j=1}^{n} d_j^+, \quad \rightarrow \min,$$

$z_0 = 0$ no job is overdue

$z_0 > 0$ at least one job is overdue

$$C_j \leq F_{\max} \qquad j = 1,2,\dots,n,$$

$$C_i + t_j \leq C_j + M(1 - x_{ij}) \qquad i,j = 1,2,\dots,n, i \neq j,$$

$$x_{ij} + x_{ji} = 1 \qquad i,j = 1,2,\dots,n, i \neq j,$$

$$C_j - d_j^+ + d_j^- = d_j \qquad j = 1,2,\dots,n,$$

$$C_j \geq t_j \qquad j = 1,2,\dots,n,$$

$$d_j^+, d_j^- \geq 0 \qquad j = 1,2,\dots,n,$$

$$x_{ij} \in \{0,1\} \qquad i,j = 1,2,\dots,n.$$

Models of management of production systems

ŠKODA AUTO University

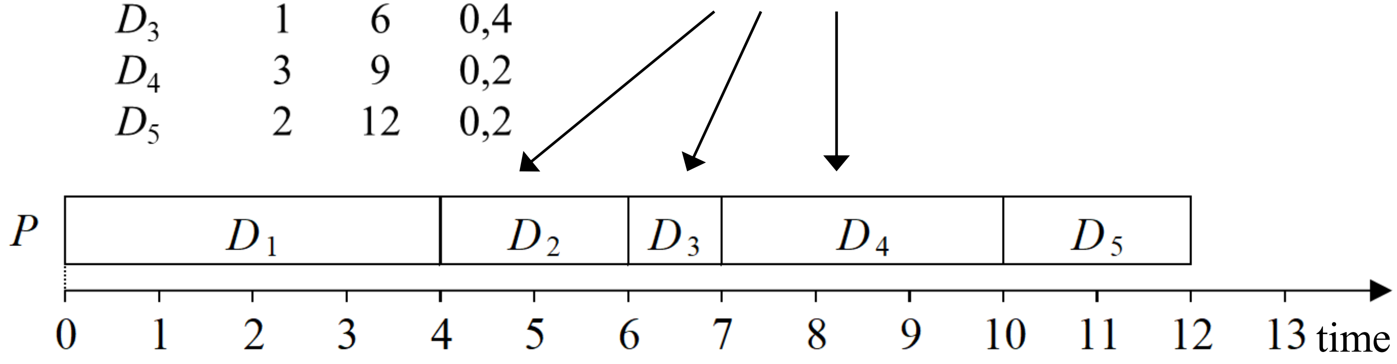
Production scheduling

- Single machine scheduling models
  - Model $1||F_{max}$ – continued
    - Example
      - Consider 5 jobs for which you know their processing time (in hours), their due time (in hours) and their relative importance in terms of weight (see table). The ready times for all jobs are zero.

Job	t_j	d_j	w_j
D_1	4	8	0,1
D_2	2	3	0,1
D_3	1	6	0,4
D_4	3	9	0,2
D_5	2	12	0,2

$F_{max} = 12$ hours

Total tardiness of jobs $= 5$ hours



Models of management of production systems

Production scheduling

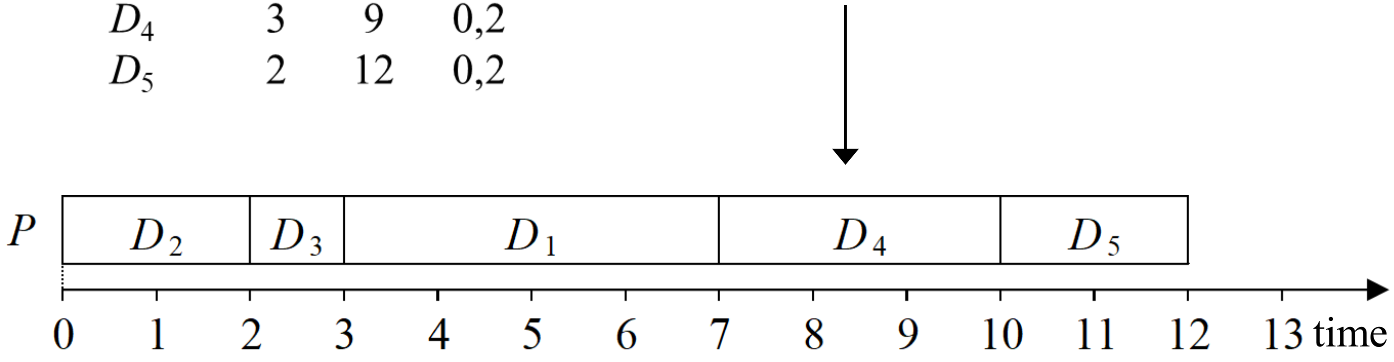
- Single machine scheduling models
  - Model $1||F_{max}$ – continued
    - Example – continued

Alternative optimal schedule

Job	t_j	d_j	w_j
D_1	4	8	0,1
D_2	2	3	0,1
D_3	1	6	0,4
D_4	3	9	0,2
D_5	2	12	0,2

$F_{max} = 12$ hours

Total tardiness of jobs $= 1$ hour



Models of management of production systems

Production scheduling

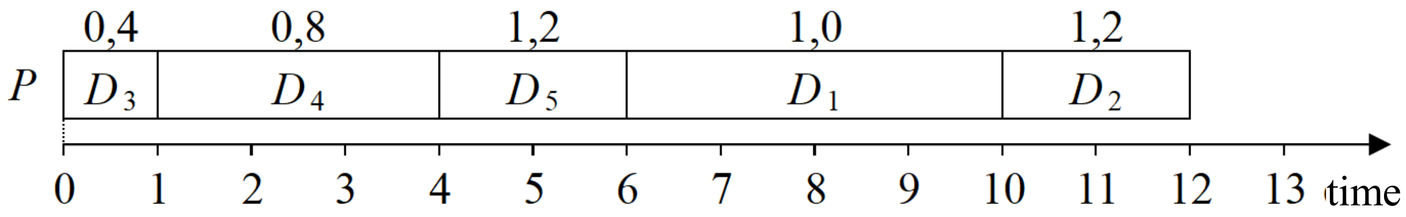
- Single machine scheduling models
  - Model $1||F_{\max}(w)$
    - In the optimal schedule, the jobs are ordered in non-increasing sequence of their weights:

$$w_{[1]} \geq w_{[2]} \geq \cdots \geq w_{[n]}.$$

    - Example – continued

Job	t_j	d_j	w_j
D_1	4	8	0,1
D_2	2	3	0,1
D_3	1	6	0,4
D_4	3	9	0,2
D_5	2	12	0,2

$$F_{\max}(w) = 1{,}2 \text{ hour}$$



Models of management of production systems

ŠKODA AUTO University

Production scheduling

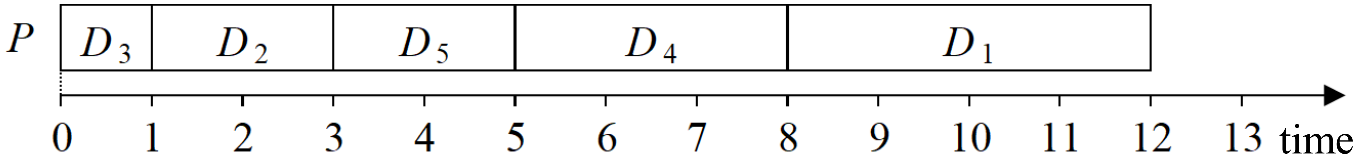
- Single machine scheduling models
  - Model $1||\bar{F}$
    - The following property gives the optimal schedule:

$$t_{[1]} \leq t_{[2]} \leq \cdots \leq t_{[n]}.$$

    - Example – continued

Job	t_j	d_j	w_j
D_1	4	8	0,1
D_2	2	3	0,1
D_3	1	6	0,4
D_4	3	9	0,2
D_5	2	12	0,2

$\bar{F}$= 5,8 hour



Models of management of production systems

Production scheduling

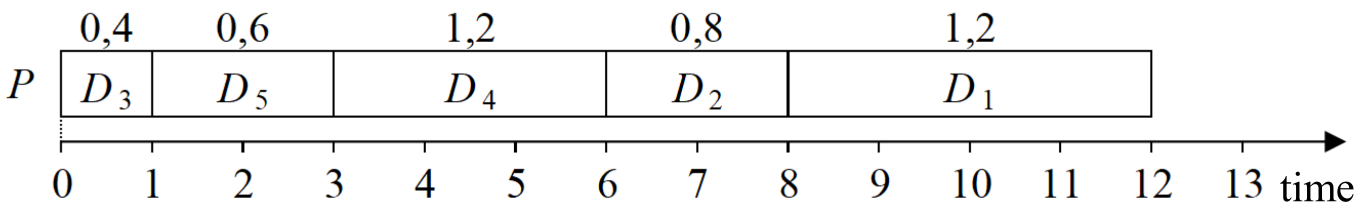
- Single machine scheduling models
  - Model $1||\bar{F}(w)$
    - The following property gives the optimal schedule:

$$\frac{t_{[1]}}{w_{[1]}} \leq \frac{t_{[2]}}{w_{[2]}} \leq \cdots \leq \frac{t_{[n]}}{w_{[n]}}.$$

    - Example – continued

Job	t_j	d_j	w_j
D_1	4	8	0,1
D_2	2	3	0,1
D_3	1	6	0,4
D_4	3	9	0,2
D_5	2	12	0,2

$\bar{F}(w)$= 4,2 hour



Models of management of production systems

ŠKODA AUTO University

Production scheduling

- Single machine scheduling models
  - Models $1||L_{\max}$ and $1||T_{\max}$
    - In the optimal schedule, the jobs are ordered in non-decreasing sequence of their due times:

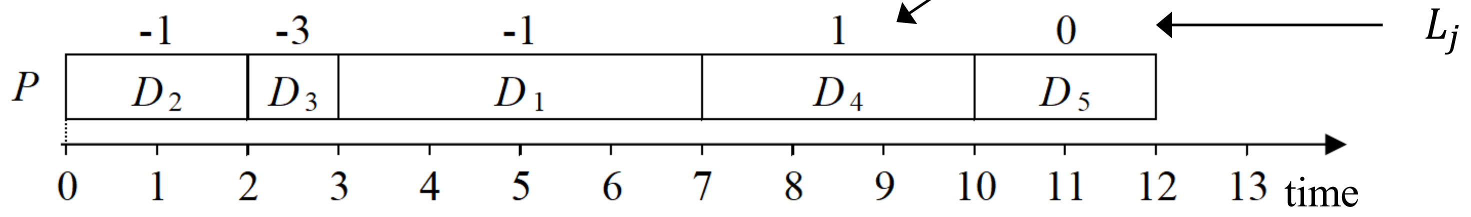
$$d_{[1]} \leq d_{[2]} \leq \cdots \leq d_{[n]}.$$

    - Example – continued

Job	t_j	d_j	w_j
D_1	4	8	0,1
D_2	2	3	0,1
D_3	1	6	0,4
D_4	3	9	0,2
D_5	2	12	0,2

$$L_{\max} = T_{\max} = 1 \text{ hour}$$

tardy job



Models of management of production systems

ŠKODA AUTO University

Production scheduling

- Single machine scheduling models
  - Models $1||L_{\max}(w)$ and $1||T_{\max}(w)$
    - Lawler's algorithm (optimization approach)
      - Step 1
        - Calculate a value representing the total processing time of all jobs: $t = \sum_{j=1}^{n} t_j$.
        - Set of unscheduled jobs is $\boldsymbol{U} = \{1,2,\ldots,n\}$.
      - Step 2
        - This step is about finding a job that will be scheduled to end at time t.
        - For this purpose, first calculate value f_j for all jobs not yet scheduled:

$$f_j = w_j(t - d_j), j \in \boldsymbol{U}, \text{ for criterion } L_{\max}(w),$$

$$f_j = w_j \max(0, t - d_j), j \in \boldsymbol{U}, \text{ for criterion } T_{\max}(w),$$

        - From all values calculated in this way, find the minimum value that corresponds to job k:

$$f_k = \min_{j \in \boldsymbol{U}} f_j \, .$$

Models of management of production systems

Production scheduling

- Single machine scheduling models
  - Models $1||L_{\max}(w)$ and $1||T_{\max}(w)$ – continued
    - Lawler's algorithm – continued
      - Step 3
        - Job k is scheduled to end at time t, tj. $C_k = t$.
        - Modify the value of t and the set of unscheduled jobs:

$$t = t - t_k, \boldsymbol{U} = \boldsymbol{U} \setminus \{k\}.$$

        - If $t = 0$ or also $\boldsymbol{U} = \emptyset$ then the optimal schedule is found, otherwise go to step 2.

Models of management of production systems

ŠKODA AUTO University

Production scheduling

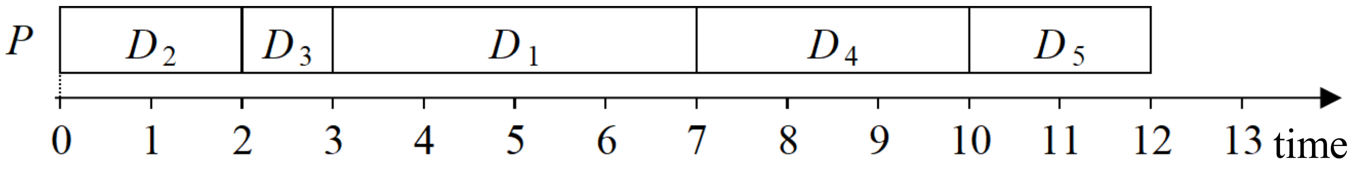
- Single machine scheduling models
  - Models $1||L_{\max}(w)$ and $1||T_{\max}(w)$ – continued
    - Example – continued

Job	t_j	d_j	w_j
D_1	4	8	0,1
D_2	2	3	0,1
D_3	1	6	0,4
D_4	3	9	0,2
D_5	2	12	0,2

Use of Lawler's algorithm for model $1||T_{\max}(w)$

	D_1	D_2	D_3	D_4	D_5	k
t_j	4	2	1	3	2	-
d_j	8	3	6	9	12	-
w_j	0,1	0,1	0,4	0,2	0,2	-
$f_j(12)$	0,4	0,9	2,4	0,6	0	5
$f_j(10)$	0,2	0,7	1,6	0,2	-	4
$f_j(7)$	0	0,4	0,4	-	-	1
$f_j(3)$	-	0	0	-	-	3
$f_j(2)$	-	0	-	-	-	2
C_j	7	2	3	10	12	-
L_j	-1	-1	-3	1	0	-
T_j	0	0	0	1	0	-

$$T_{\max}(w) = 0{,}2 \text{ hours}$$



Models of management of production systems

Production scheduling

- Single machine scheduling models
  - Model $1||\bar{L}$
    - The following property gives the optimal schedule:

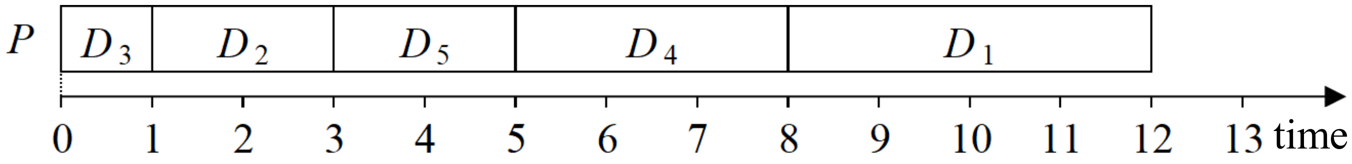
$$t_{[1]} \leq t_{[2]} \leq \cdots \leq t_{[n]}.$$

    - Example – continued

Job	t_j	d_j	w_j
D_1	4	8	0,1
D_2	2	3	0,1
D_3	1	6	0,4
D_4	3	9	0,2
D_5	2	12	0,2

	D_1	D_2	D_3	D_4	D_5
t_j	4	2	1	3	2
d_j	8	3	6	9	12
w_j	0,1	0,1	0,4	0,2	0,2
C_j	12	3	1	8	5
L_j	4	0	-5	-1	-7

$\bar{L}$= -1,8 hour



Models of management of production systems

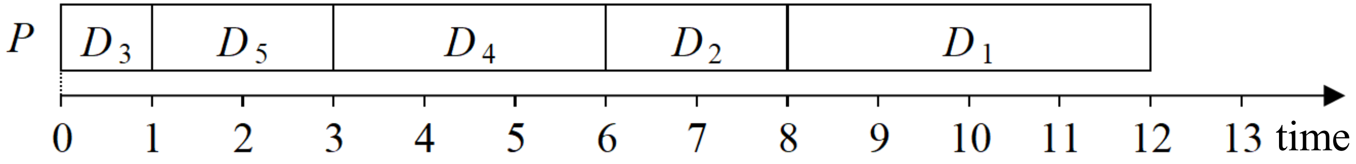
ŠKODA AUTO University

Production scheduling

- Single machine scheduling models
  - Model $1||\bar{L}(w)$
    - The following property gives the optimal schedule : $$\frac{t_{[1]}}{w_{[1]}} \leq \frac{t_{[2]}}{w_{[2]}} \leq \cdots \leq \frac{t_{[n]}}{w_{[n]}}.$$
    - Example – continued

Job	t_j	d_j	w_j
D_1	4	8	0,1
D_2	2	3	0,1
D_3	1	6	0,4
D_4	3	9	0,2
D_5	2	12	0,2

	D_1	D_2	D_3	D_4	D_5
t_j	4	2	1	3	2
d_j	8	3	6	9	12
w_j	0,1	0,1	0,4	0,2	0,2
C_j	12	8	1	6	3
L_j	4	5	-5	-3	-9
w_jL_j	0,4	0,5	-2,0	-0,6	-1,8



$\bar{L}(w) =$ -3,5 hour

Models of management of production systems

Production scheduling

- Single machine scheduling models
  - Model $1||N$
    - Moore's algorithm (optimization approach)
      - Step 1
        - Make the schedule using the following property:
          $$d_{[1]} \leq d_{[2]} \leq \cdots \leq d_{[n]}.$$
        - Jobs in this order are sequenced into $\boldsymbol{E}$. For overdue jobs the following set is introduced: $\boldsymbol{L} = \emptyset$.
      - Step 2
        - If sequence $\boldsymbol{E}$ does not contain any overdue job, then sequence $\boldsymbol{E}$ extended by jobs from set $\boldsymbol{L}$ (in any order of jobs) generates optimal schedule $\boldsymbol{R}$.
        - Number of overdue jobs is $N = |\boldsymbol{L}|$, algorithm ends.

Models of management of production systems

Production scheduling

- Single machine scheduling models
  - Model $1||N$ – continued
    - Moore's algorithm – continued
      - Step 3
        - In sequence $\boldsymbol{E}$, find the first overdue job. Let it be job $D_{[k]}$ on the place k of the order.
        - Among the first k jobs, find job $D_{[m]}$ with the longest processing time $t_{[m]}$:

$$t_{[m]} = \max_{j=1,2,\ldots,k} t_{[j]}\,.$$

        - Move job $D_{[m]}$ from sequence $\boldsymbol{E}$ to set $\boldsymbol{L}$ and goto step 2.

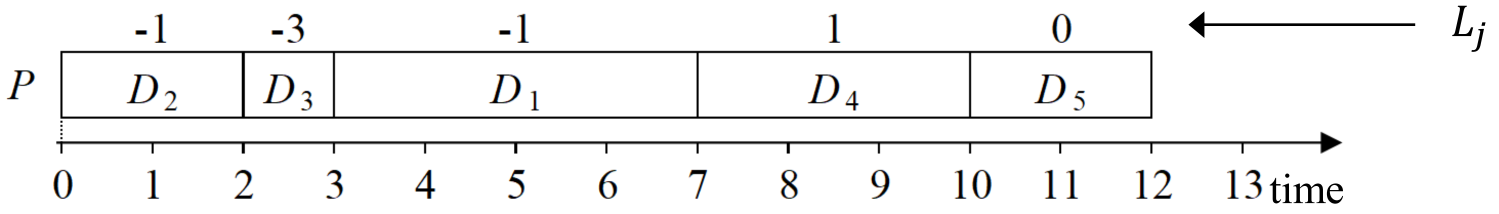
Models of management of production systems

Production scheduling

- Single machine scheduling models
  - Model $1||N$ – continued
    - Example – continued

$\boldsymbol{E} = (D_2, D_3, D_1, D_4, D_5)$

Job	t_j	d_j	w_j
D_1	4	8	0,1
D_2	2	3	0,1
D_3	1	6	0,4
D_4	3	9	0,2
D_5	2	12	0,2



- The first (and the only one) overdue job is D_4, which is on the 4th palce in sequence $\boldsymbol{E}$. Among the first 4 jobs, job D_1 has the longest processing time, therefore it is erased from sequence $\boldsymbol{E}$ and it is added to set $\boldsymbol{L}$. Because in modified sequence $\boldsymbol{E}$ no job is overdue anymore, the algorithm ends.

$\boldsymbol{R} = (D_2, D_3, D_4, D_5, D_1)$,
$N = 1$.

Models of management of production systems

ŠKODA AUTO University

Production scheduling

- Extensions of basic single machine scheduling model
  - Model $1|r_{j;}prmp|\bar{T}$
    - Ready times of jobs do not have to be zero (release of condition S_1).
    - Jobs can be interrupted.
    - Example
      - Consider 3 jobs for which we know the processing times, ready times and due times (all values are in hours).

Job	t_j	r_j	d_j
D_1	4	1	7
D_2	2	0	4
D_3	1	3	5

Models of management of production systems

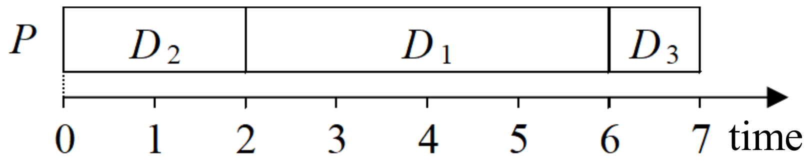
ŠKODA AUTO University

Production scheduling

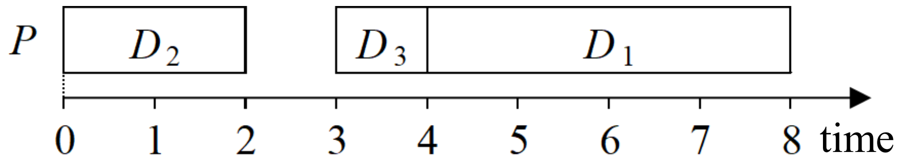
- Extensions of basic single machine scheduling model
  - Model $1|r_j;prmp|\bar{T}$ – continued
    - Example – continued

Job	t_j	r_j	d_j
D_1	4	1	7
D_2	2	0	4
D_3	1	3	5

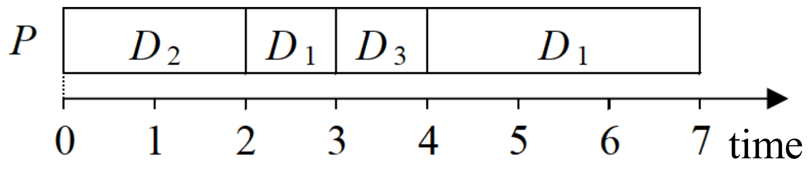
      - Schedule without an interruption, $\bar{T} = 2/3$ hour



      - Schedule without an interruption, $\bar{T} = 1/3$ hour



      - Schedule with the interruption, $\bar{T} = 0$ hour



Models of management of production systems

Production scheduling

- Extensions of basic single machine scheduling model
  - Model $1|prec|T_{\max}$
    - Technological dependence of jobs, expressed by means of precedence relations (release of condition S_2).
    - Extension of Lawler's algorithm (optimization approach).
      - Step 1
        - Calculate a value representing the total processing time of all jobs: $t = \sum_{j=1}^{n} t_j$.
        - Set of unscheduled jobs is $\boldsymbol{U} = \{1,2,\dots,n\}$.
      - Step 2
        - This step is about finding a job that will be scheduled to end at time t.
        - Let set $\boldsymbol{V} \subseteq \boldsymbol{U}$ contain jobs that have no successors or which successors are already scheduled, i.e. there is no job $i \in \boldsymbol{U}$, which is their successor.
        - For all jobs in set $\boldsymbol{V}$ then calculate value $f_j = \max(0, t - d_j), j \in \boldsymbol{V}$.
        - From all values calculated in this way, find the minimum value that corresponds to job k:

$$f_k = \min_{j \in \boldsymbol{V}} f_j \, .$$

Models of management of production systems

ŠKODA AUTO University

Production scheduling

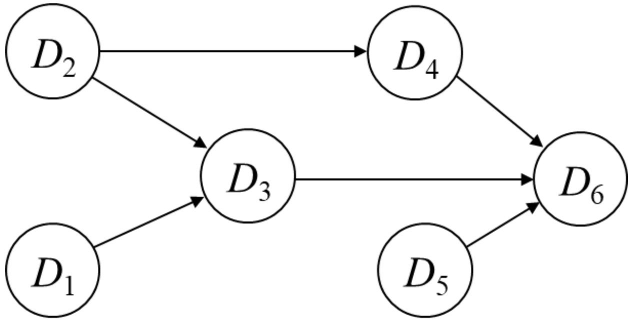
- Extensions of basic single machine scheduling model
  - Model $1|prec|T_{\max}$ – continued
    - Extension of Lawler's algorithm – continued
      - Step 3
        - Job k is scheduled to end at time t, tj. $C_k = t$.
        - Modify the value of t and the set of unscheduled jobs:
          $$t = t - t_k, \boldsymbol{U} = \boldsymbol{U} \setminus \{k\}.$$
        - If $t = 0$ or also $\boldsymbol{U} = \emptyset$ then the optimal schedule is found, otherwise go to step 2.

Models of management of production systems

ŠKODA AUTO University

Production scheduling

- Extensions of basic single machine scheduling model
  - Model $1|prec|T_{\max}$ – continued
    - Example
      - Make a schedule for 6 jobs, given by the precedence graph. The table shows the processing times of the jobs and their due times (both values are in hours).



	D_1	D_2	D_3	D_4	D_5	D_6
t_j	3	2	5	4	6	7
d_j	5	8	16	12	20	25

Optimal schedule: $(D_2, D_1, D_4, D_3, D_5, D_6)$

	D_1	D_2	D_3	D_4	D_5	D_6
C_j	5	2	14	9	20	27
T_j	0	0	0	0	0	2

$T_{\max} = 2$ hour

Models of management of production systems

ŠKODA AUTO University

Production scheduling

- Extensions of basic single machine scheduling model
  - Model $1|s_{jk}|C_{\max}$
    - The setup times of jobs depends on their order in the schedule (release of condition S_3):

      s_{jk} setup time for start of processing job k immediately following realization of job j.
    - The problem of finding the optimal schedule can be transformed into the problem of finding the jobs order that minimizes the total time of all setups.
    - Analogy with the travelling salesman problem, in which distances between locations correspond to setup times.

$$x_{jk} = \begin{cases} 1 & \text{if } j\text{-th job is processed immediately before } k\text{-th job} \\ 0 & \text{otherwise,} \end{cases} \qquad \begin{array}{l} j = 1,2,\dots,n, \\ k = 1,2,\dots,n. \end{array}$$

Models of management of production systems

ŠKODA AUTO University

Production scheduling

- Extensions of basic single machine scheduling model
  - Model $1|s_{jk}|C_{\max}$ – continued
    - Mathematical model

$$z = \sum_{j=1}^{n}\sum_{k=1}^{n} s_{jk}x_{jk} + \sum_{j=1}^{n} t_j \quad \rightarrow \quad \min,$$

$$\sum_{k=1}^{n} x_{jk} = 1 \qquad j = 1,2,\dots,n,$$

$$\sum_{j=1}^{n} x_{jk} = 1 \qquad k = 1,2,\dots,n,$$

$$u_j + 1 - (n-1)(1 - x_{jk}) \le u_k \qquad \begin{matrix} j = 1,2,\dots,n, \\ k = 2,3,\dots,n, \end{matrix}$$

$$x_{jk} \in \{0,1\} \qquad \begin{matrix} j = 1,2,\dots,n, \\ k = 1,2,\dots,n, \end{matrix} \qquad u_j \ge 0 \qquad j = 1,2,\dots,n.$$

Models of management of production systems

ŠKODA AUTO University

Production scheduling

- Extensions of basic single machine scheduling model
  - Model $1|s_{jk}|C_{\max}$ – continued
    - Nearest neighbor algorithm (heuristic method)
      - Step 1
        - Select any job (suppose the processing begins and ends with job 1), denote it as i. This job is included as the first job in the schedule, denoted as sequence $\boldsymbol{R}$. In this step, therefore $\boldsymbol{R} = (1)$.
        - Total processing time of all jobs including setup times is denoted as t. Set $t = t_i = t_1$. All other jobs create a set of not yet scheduled jobs $\boldsymbol{U} = \{2,3, \dots, n\}$.
      - Step 2
        - Find (among the jobs not yet scheduled) to job i its "nearest neighbor", which is job k with the minimal setup time:

$$s_{ik} = \min_{j \in \boldsymbol{U}} s_{ij} \, .$$

Models of management of production systems

ŠKODA AUTO University

Production scheduling

- Extensions of basic single machine scheduling model
  - Model $1|s_{jk}|C_{\max}$ – continued
    - Nearest neighbor algorithm
      - Step 3
        - Include job k at the end of sequence $\boldsymbol{R}$ and exclude it form set $\boldsymbol{U}$. Modify the value of t and replace index i with index k:

$$\boldsymbol{R} = \boldsymbol{R} + (\boldsymbol{k}), \boldsymbol{U} = \boldsymbol{U} \setminus \{k\}, t = t + s_{ik} + t_k, i = k.$$

        - If $\boldsymbol{U} = \emptyset$, go to step 4, otherwise go to step 2.
      - Step 4
        - Since, by assumption, the whole schedule ends by setup of the machine to job 1, modify the value of t as follows:

$$t = t + s_{k1}.$$

        - Algorithm ends, sequence $\boldsymbol{R}$ corresponds to the feasible schedule, the value of t equals to the total processing time of all jobs including all setup times in the schedule.

Models of management of production systems

ŠKODA AUTO University

Production scheduling

- Extensions of basic single machine scheduling model
  - Model $1|s_{jk}|C_{\max}$ – continued
    - Example
      - The company wants to schedule 8 jobs on 1 machine so that they are completed in minimum time. The table shows the job processing times and the setup times between jobs (all values are in minutes).

	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	t_j
D_1	0	15	17	12	14	18	16	10	50
D_2	12	0	13	24	20	13	15	14	40
D_3	14	18	0	15	19	11	10	17	30
D_4	13	14	17	0	18	11	16	15	50
D_5	21	17	18	17	0	15	12	12	60
D_6	12	14	13	14	20	0	16	11	20
D_7	11	22	17	19	15	14	0	13	60
D_8	10	13	17	16	13	12	15	0	40

Nearest neighbor method:
$\boldsymbol{R} = (1,8,6,3,7,5,2,4)$,
$t = 464$ min.

Optimal schedule:
$\boldsymbol{R} = (1,4,6,2,3,7,5,8)$,
$t = 447$ min.

Models of management of production systems

ŠKODA AUTO University

Production scheduling

- Parallel machine models
  - Model $P_m|prmp|F_{\max}$
    - There are m identical processors arranged in parallel in the system.
    - There are n jobs processed, each consists of one operation, there are no precedence relations between jobs.
    - Ready times of all jobs are zero.
    - Jobs can be interrupted after finishing the realized operation and can continue on the same or another machine.
    - Notation "$[i] = j$" means that j-th job is processed in i-th place in order of the schedule.
    - For the minimal makespan it is valid:

average occupancy time of machine

maximal processing time

$$F_{\max} = \max\left\{\frac{1}{m}\sum_{j=1}^{n} t_j;\ \max_{j=1,2,\dots,n} t_j\right\}.$$

Models of management of production systems

Production scheduling

- Parallel machine models
  - Model $P_m|prmp|F_{\max}$ – continued
    - McNaughton‘s algorithm (optimization approach)
      - Step 1
        - Calculate the value of $F_{\max}$. Assign job D_1 to machine P_1 at time 0.
      - Step 2
        - Assign (to the same machine) other jobs according to their ordinal numbers respecting $F_{\max}$.
        - Three cases can occur:

          a) All remaining jobs can be assigned to the machine, i.e. the last job assigned is job D_n.

          b) Last assigned job D_k is completed exactly on time $F_{\max}$.

          c) When job D_k is assigned, the value of $F_{\max}$ will be exceeded.

Models of management of production systems

Production scheduling

- Parallel machine models
  - Model $P_m|prmp|F_{\max}$ – continued
    - McNaughton‘s algorithm – continued
      - Step 3
        - Next, we proceed according to which case occurred in the previous step:

          a) All jobs are scheduled, the algorithm ends.

          b) Assign job D_{k+1} to the next machine at time 0 and go to step 2.

          c) Split job D_k so that its first part will be completed on the scheduled machine at time $F_{\max}$ and assign its remaining part to the next machine at time 0, go to step 2.

      - This procedure can lead to a maximum of $m-1$ interruptions.

Models of management of production systems

ŠKODA AUTO University

Production scheduling

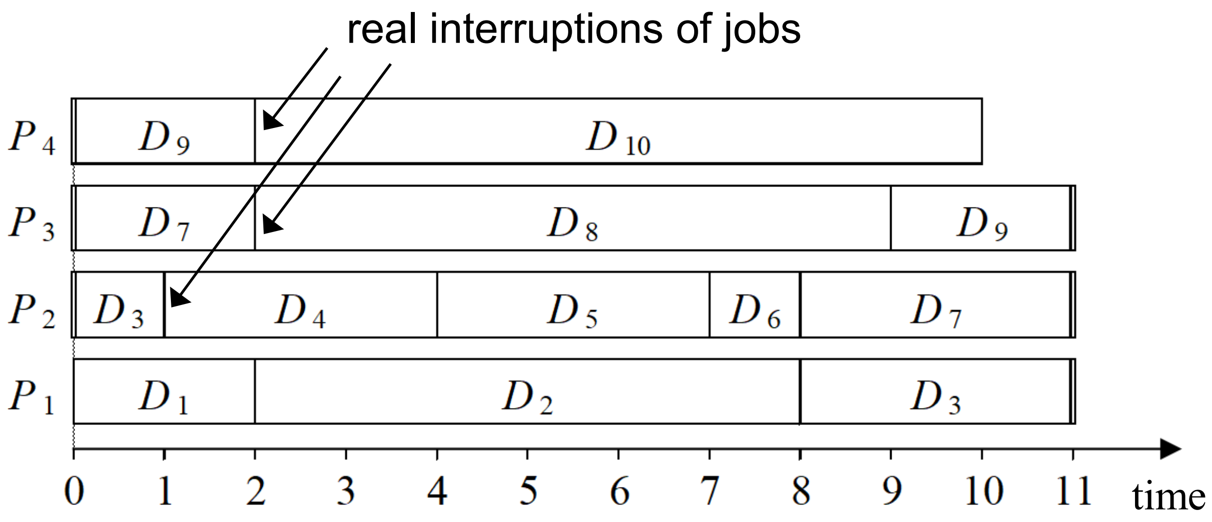
- Parallel machine models
  - Model $P_m|prmp|F_{\max}$ – continued
    - Example
      - The problem is to schedule 10 jobs on 4 machines so that they are processed in minimum time, i.e. the makespan will be minimal. The table shows the processing times of the jobs (in min). The operations that make up each job have a duration of 1 min, so all jobs can be interrupted at any time after each minute of processing.

	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9	D_{10}
t_j	2	6	4	3	3	1	5	7	4	8

Models of management of production systems

Production scheduling

- Parallel machine models
  - Model $P_m|prmp|F_{\max}$ – continued
    - Example – continued
      - Calculate $F_{\max} = \max\left\{\frac{43}{4}; 8\right\} = \frac{43}{4}$
      - Since the jobs can only be interrupted after the completion of the operation currently being performed, it is necessary to calculate the ceiling, i.e. 11 min, from the value 43/4.



Models of management of production systems

ŠKODA AUTO University

Production scheduling

- Parallel machine models
  - Model $P_m|prmp|F_{\max}$ – continued
    - Mathematical model for obtaining the schedule without the interruption of jobs (if it exists)

$$x_{ij} = \begin{cases} 1 & \text{if } j\text{-th job is scheduled on } i\text{-th machine} \\ 0 & \text{otherwise,} \end{cases} \qquad i = 1,2,\dots,m, \quad j = 1,2,\dots,n.$$

$$z = \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} \;\rightarrow\; \max,$$

The objective is not necessary to obtain a feasible schedule

$$\sum_{j=1}^{n} x_{ij} t_j \le F_{\max} \qquad i = 1,2,\dots,m,$$

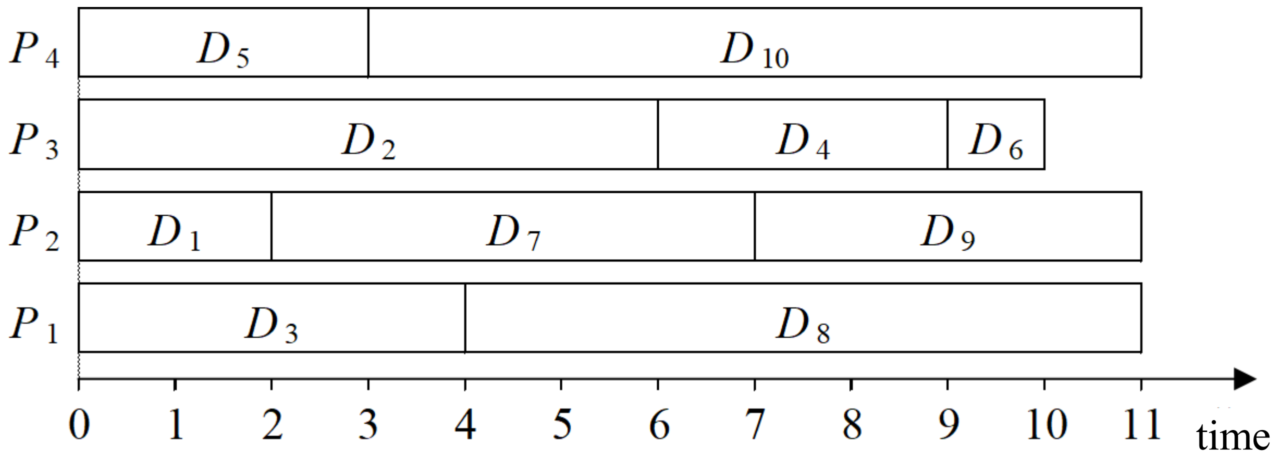
$$\sum_{i=1}^{m} x_{ij} = 1 \qquad j = 1,2,\dots,n.$$

Models of management of production systems

ŠKODA AUTO University

Production scheduling

- Parallel machine models
  - Model $P_m|prmp|F_{\max}$ – continued
    - Example – continued



Models of management of production systems

Production scheduling

- Parallel machine models
  - Model $P_m||F_{\max}$
    - Jobs cannot be interrupted.
    - Generalization of the mathematical model formulated for the model $P_m|prmp|F_{\max}$
      - $F_{\max}$ is a variable, not a calculated constant, it is necessary to set integrality constraints (due to the continuity of operations in the job).
      - The objective is $z = F_{max} \rightarrow \min.$
      - Jobs cannot be interrupted.
    - NP-hard problem.

Models of management of production systems

Production scheduling

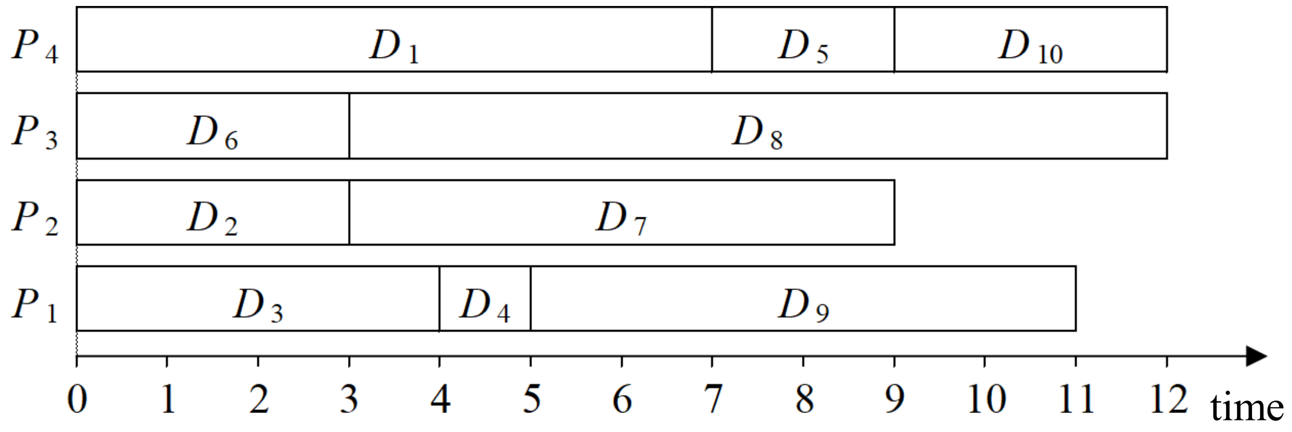
- Parallel machine models
  - Model $P_m||F_{\max}$ – continued
    - Example
      - The problem is to schedule 10 jobs on 4 machines so that they are processed in minimum time, i.e. the makespan will be minimal. The table shows the processing times of the jobs (in min). The operations that make up each job have a duration of 1 min, so all jobs can be interrupted at any time after each minute of processing.

	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9	D_{10}
t_j	7	3	4	1	2	3	6	9	6	3

Models of management of production systems

Production scheduling

- Parallel machine models
  - Model $P_m||F_{\max}$ – continued
    - Example – continued



Models of management of production systems

ŠKODA AUTO University

Production scheduling

- Parallel machine models
  - Model $P_m||F_{\max}$ – continued
    - Heuristic algorithm:
      - Step 1
        - Make the schedule using the following property:

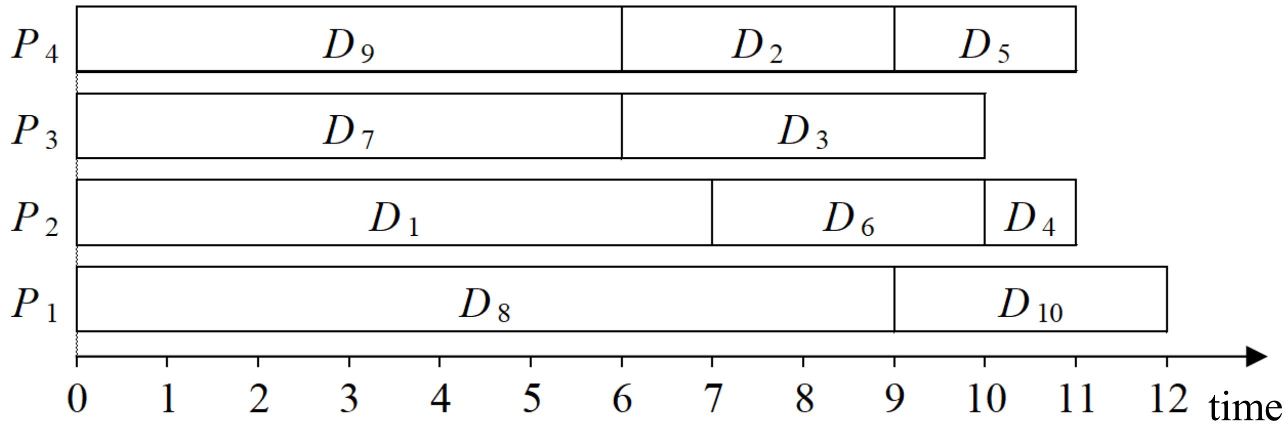
$$t_{[1]} \geq t_{[2]} \geq \cdots \geq t_{[n]}.$$

      - Step 2
        - Select first m jobs in the sequence from step 1 and assign them to machines at time 0.
      - Step 3
        - Select next job in the sequence and assign it after the last job to the machine, which is released first, i.e. after the job, which of all the last jobs on the machines ends first.
        - Repeat this process until all jobs are scheduled.

Models of management of production systems

Production scheduling

- Parallel machine models
  - Model $P_m||F_{\max}$ – continued
    - Example – continued



Models of management of production systems

Production scheduling

- Parallel machine models
  - Model $P_m||\bar{F}$
    - The same principle as the optimization procedure for the model $1||\bar{F}$:
      - Step 1
        - Make the schedule using the following property :

$$t_{[1]} \leq t_{[2]} \leq \cdots \leq t_{[n]}.$$

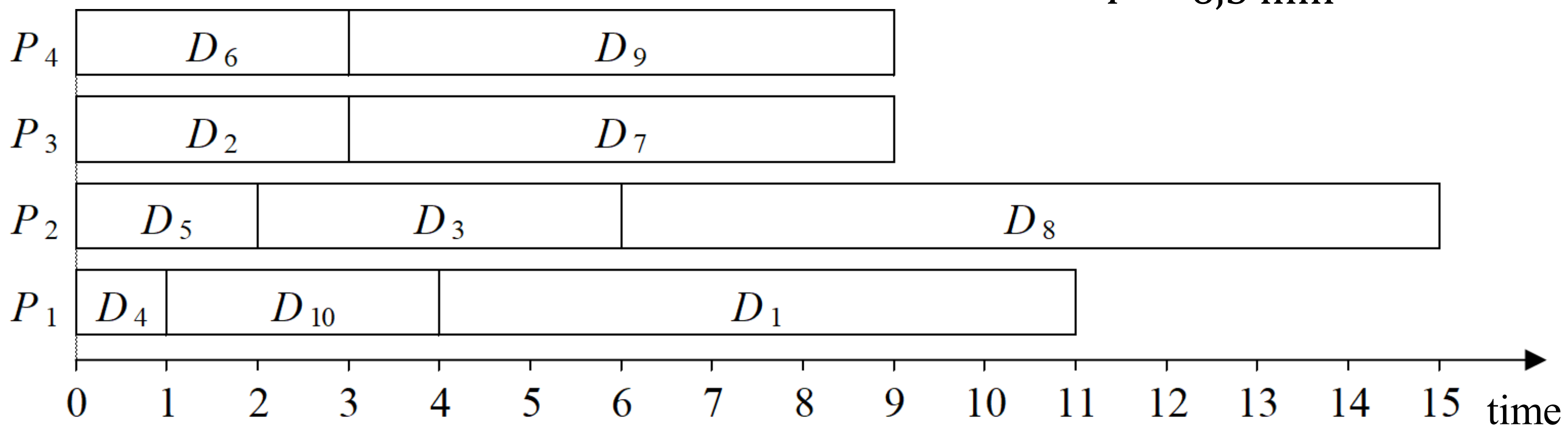
      - Step 2
        - Select first m jobs in the sequence from step 1 and assign them to machines at time 0.
      - Step 3
        - Select next job in the sequence and assign it after the last job to the machine, which is released first, i.e. after the job, which of all the last jobs on the machines ends first.
        - Repeat this process until all jobs are scheduled.

Models of management of production systems

Production scheduling

- Parallel machine models
  - Model $P_m||\bar{F}$ – continued
    - Example – continued

$\bar{F} = 6{,}3$ min



Models of management of production systems

ŠKODA AUTO University

Production scheduling

- Flow shop scheduling
  - Model $F_m||C_{\max}$
    - There are m serial machines in the system.
    - There are n jobs processed, each consists of m operations, each of them is processed on one machine.
    - The jobs must be processed on the machines in the same order, i.e. i-th operation of each job is processed on i -th machine.
    - Processing time of i-th operation of j-th job is denoted as t_{ij}.
    - No machine can process more than one job at a time, and no job can be processed on multiple machines simultaneously.

Models of management of production systems

Production scheduling

- Flow shop scheduling
  - Model $F_m||C_{\max}$ – continued
    - Constraints:
      - S_1 : Ready times for all jobs and machines are 0.
      - S_2 : There are no precedence relations between jobs.
      - S_3 : Processing times of jobs are independent of the order of their processing on the machine.
      - S_4 : Operations cannot be interrupted.

Models of management of production systems

ŠKODA AUTO University

Production scheduling

- Flow shop scheduling
  - Model $F_m||C_{\max}$ – continued
    - Example
      - Find a feasible schedule for 5 jobs that need to be scheduled on 3 serially arranged machines. The table contains the job processing times (in min) on each machine.

	D_1	D_2	D_3	D_4	D_5
t_{1j}	3	1	2	2	1
t_{2j}	1	2	1	3	2
t_{3j}	2	1	2	1	1

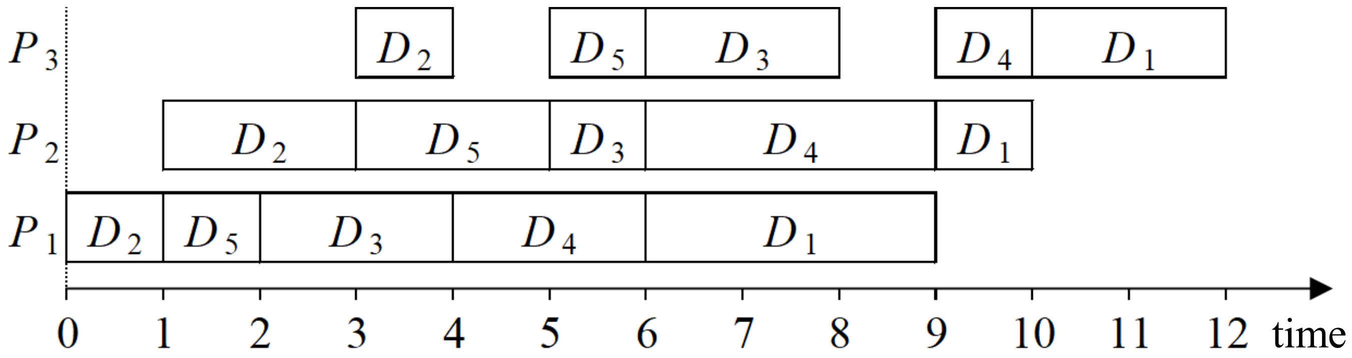
Models of management of production systems

ŠKODA AUTO University

Production scheduling

- Flow shop scheduling
  - Model $F_m||C_{\max}$ – continued
    - Example – continued
      - Feasible schedule:

	D_1	D_2	D_3	D_4	D_5
t_{1j}	3	1	2	2	1
t_{2j}	1	2	1	3	2
t_{3j}	2	1	2	1	1



Models of management of production systems

ŠKODA AUTO University

Production scheduling

- Flow shop scheduling
  - Model $F_m||C_{\max}$ – continued
    - Johnson‘s algorittm (optimization approach) for the problem with 2 machines
      - Step 1
        - Set of jobs is split to two subsets:

$$\boldsymbol{U} = \left\{D_j \middle| t_{1j} < t_{2j}\right\}, \qquad \boldsymbol{V} = \left\{D_j \middle| t_{1j} \geq t_{2j}\right\}.$$

      - Step 2
        - Order jobs in set $\boldsymbol{U}$ to the non-decreasing sequence $(\boldsymbol{U})$ of operation processing times t_{1j}.
          Order jobs in set $\boldsymbol{V}$ to the non-increasing sequence $(\boldsymbol{V})$ of operation processing times t_{2j}.
      - Step 3
        - Optimal jobs schedule $(\boldsymbol{R})$ on both machines consists of sequence $(\boldsymbol{U})$ followed by sequence $(\boldsymbol{V})$. Value $C_{\max}$ is given by the completion time of the last job in sequence $(\boldsymbol{V})$ on the second machine.

Models of management of production systems

ŠKODA AUTO University

Production scheduling

- Flow shop scheduling
  - Model $F_m||C_{\max}$ – continued
    - Example
      - Let 6 jobs be given, which must be scheduled to 2 serially arranged machines. The table shows the processing times (in min) of the jobs on both machines. The objective is to minimize the completion time of all batches.

	D_1	D_2	D_3	D_4	D_5	D_6
t_{1j}	2	1	3	3	2	2
t_{2j}	1	2	4	1	4	2

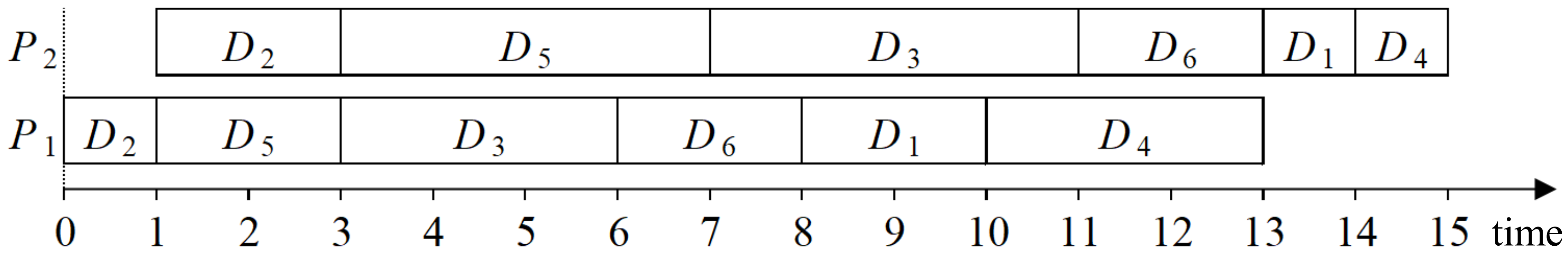
Models of management of production systems

ŠKODA AUTO University

Production scheduling

- Flow shop scheduling
  - Model $F_m||C_{\max}$ – continued
    - Example – continued
      - Step 1 $\boldsymbol{U} = \{D_2, D_3, D_5\}$, $\boldsymbol{V} = \{D_1, D_4, D_6\}$.
      - Step 2 $(\boldsymbol{U}) = (D_2, D_5, D_3)$, $(\boldsymbol{V}) = (D_6, D_1, D_4)$.
      - Step 3 $(\boldsymbol{R}) = (D_2, D_5, D_3, D_6, D_1, D_4)$, $C_{\max}$ =15.

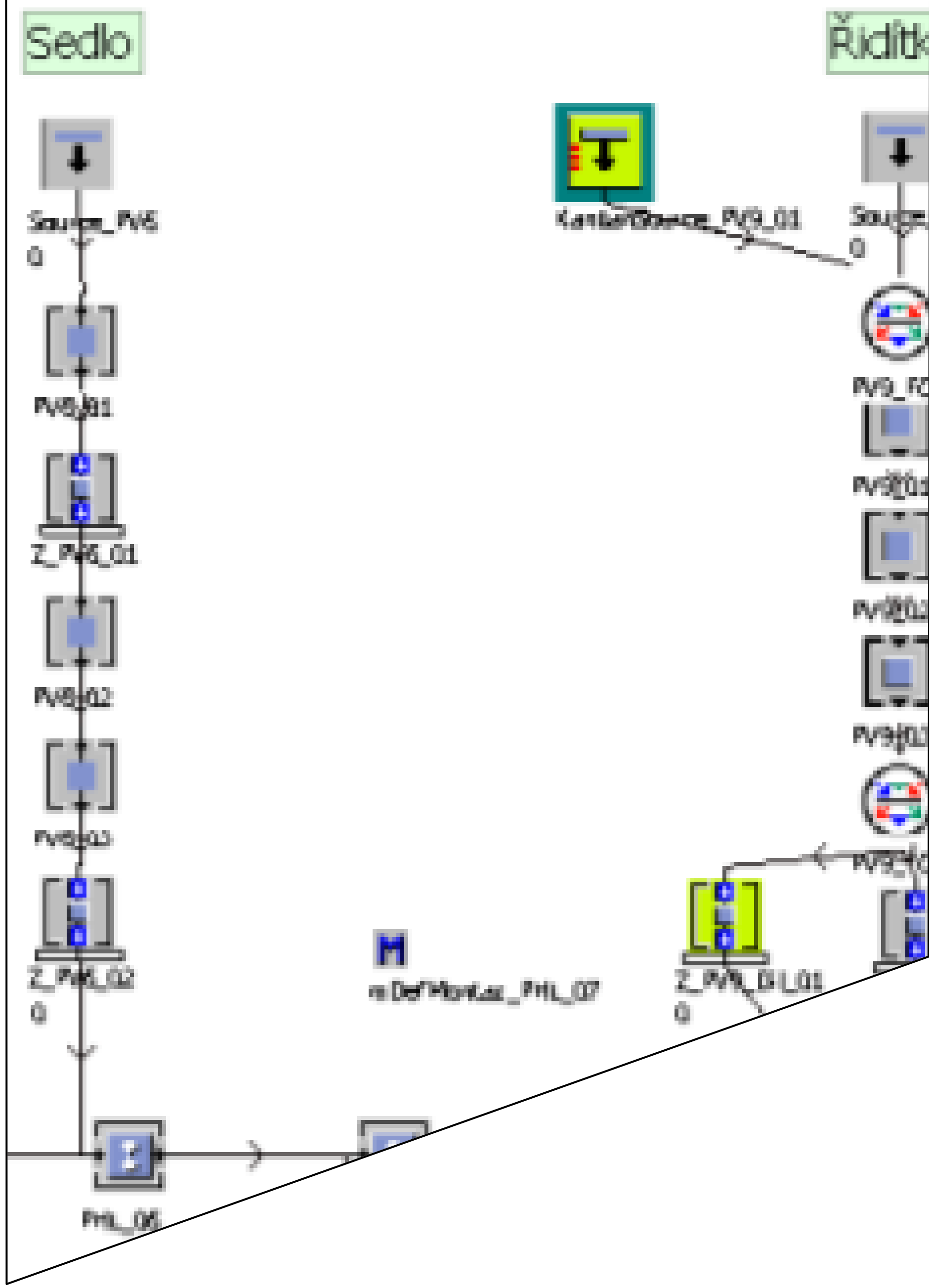
	D_1	D_2	D_3	D_4	D_5	D_6
t_{1j}	2	1	3	3	2	2
t_{2j}	1	2	4	1	4	2



ŠKODA AUTO University

4

Computer simulation



Computer simulation

ŠKODA AUTO University

Introduction to computer simulation

- Definition
    - Representation of a real (but also planned, not yet existing) system with its stochastic and dynamic processes in the form of a simulation model.
    - The basic idea is to simulate the behaviour of a real system through a simulation model and, based on experimentation with this model, to propose changes that will improve the functioning of the system.
- Areas of application of computer simulation
    - Optimization of large-scale production systems.
    - Analysis of logistics processes inside a company or within the entire supply chain.
    - Optimization of warehousing processes.
    - Optimization of production scheduling.
    - Improvement of communication systems, optimization of information flows.

Computer simulation

Introduction to computer simulation

- Costs associated with computer simulation
  - Staff costs for a qualified analyst and programmer.
  - Costs associated with managers' time spent communicating with the analyst during the project.
  - The cost of powerful computing equipment (HW).
  - Costs for software.
  - Data collection costs.

Computer simulation

Introduction to computer simulation

- Simulation project
  - Problem recognition and goals setting.
  - Creation of a conceptual model (CM).
  - Data collection and analysis.
  - Creation of a simulation model (SM).
  - Verification and validation of the model (checking whether the SM is consistent with the original CM and whether the SM is in compliance with the reality).
  - Performing experiments.
  - Analysis of results.
  - Creation of project documentation.
  - Implementation of results.

Computer simulation

ŠKODA AUTO University

Data analysis

- Basic analytical tools
  - *tables,*
  - *diagrams,*
  - *charts,*
  - *schemes*.
- General methods of processing data
  - *Clustering* – the goal is to sort the data into groups (clusters) so that the elements of each group share a common attribute (car type, car-body type, color).
  - *Filtering* – selection of elements that meet the specified conditions (simple filters, multiple filters).
  - *Sorting - sorting data according to the values of one or more attributes (the so-called key), the result is the order of elements.*
  - *Pairing - combining information (character values) from several databases or lists obtained, for example, on the basis of passing products through individual check points.*

Computer simulation

Data analysis

- Software
  - Access, Excel, SAS, SPSS, Gretl, Statgraphics, MATLAB, R.
- Statistical data analysis
  - *(Random) Experiment* – an experiment that can be repeated and the outcome of which is not known in advance.
  - *Random variable* – variable whose value is given by the result of an experiment (discrete, continuous).
  - *(Random) Event* – the result of an experiment expressed by the value of a random variable. .
  - *Probability of a random event* – numerical expression of the degree of possibility of occurrence of a random event.
  - *Probability distribution* –a rule that assigns to each value or interval of values the probability that a random variable will take on that value or a value from that interval.

Computer simulation

ŠKODA AUTO University

Data analysis

- Statistical data analysis
  - *Random number* is defined as the value of a uniform probability distribution on the interval (0, 1). Several generators have been developed to generate random numbers, the most commonly used of which are arithmetic generators. Random numbers are obtained by calculating each number using some arithmetic operation from the previous number. Since this is an arithmetic calculation and not a random operation, numbers obtained in this way can only be described as pseudo-random numbers. The random numbers generated are then transformed by various methods into values of random variables.
  - *The inverse transformation method* is one of the methods used to convert a random number r into values of a random variable X. Example for values of a uniform probability distribution on the interval (a, b):

$$F(x) = \frac{x-a}{b-a}$$ cumulative distribution function on the interval (a, b),

$$r = \frac{x-a}{b-a}$$ random number,

$$x = a + r(b-a)$$ value of random variable with uniform probability distribution.

ŠKODA AUTO University

Thank you for attention

Jan Fábry

Department of Production, Logistics and Quality Management

fabry@savs.cz www.janfabry.cz

www.savs.cz