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Modeling of Production and Logistics Systems

Exercises

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1/30/2022

Examples for MPLS exercises

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Repetition of linear and integer programming

- Example 1 – Cutting stock problem
  - Firm produces garden laths fence. There are only standard laths 200 cm long at disposal at storehouse. To produce a fence, the firm needs exactly 1200 laths 80 cm long, 3100 laths 50 cm long and 500 laths 30 cm long. You have to design a cutting plan to minimize total amount of laths 200 cm long that were cut. Formulate a mathematical model of the problém, solve the problem in the solver of MS Excel and MPL for Windows.
- Example 2 – Disjunctive constraints (either-or constraints)
  - Three products can be produced on a machine either in the sequence $P_1 \rightarrow P_2 \rightarrow P_3$ or $P_3 \rightarrow P_2 \rightarrow P_1$. Assume production of P_i takes t_i. Formulate the constraints for allowable production.
- Example 3 – Production level planning with Yes-or-No decision
  - A company is considering whether to produce a new product or not. If so, the level of production should be at least 500 units but not more than 1000 units. Formulate the decision constraints.
- Example 4 – Planning production on discrete levels
  - A company decides to produce either 500 or 1000 or 2000 units of certain product. Formulate the decision constraints.

Examples for MPLS exercises

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Repetition of linear and integer programming

- Example 5 – Disjunctive variables (k out of n variables must be positive)
  - A company is able to produce n types of products. It is considering to produce only k of them. For each product i, maximal production level q_i is given. Formulate the decision constraints.

- Example 6 – Bottleneck assignment problem
  - The project consists of 5 independent parts. In the company 5 departments can manage the parts individually. Historical data shows average times (in days) departments finished similar tasks (see the table). N.A. represents the fact a department did not work on such task in the past. The company wants to finish the whole project as soon as possible. Formulate a mathematical model of the problem and solve the problem in the solver of MS Excel and MPL for Windows

Time	Part1	Part2	Part3	Part4	Part5
Dept1	25	15	N.A.	17	25
Dept2	22	N.A.	22	20	22
Dept3	20	18	25	16	23
Dept4	N.A.	20	30	21	28
Dept5	27	19	27	18	N.A.

Examples for MPLS exercises

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Product designing

- Example 7 – Collaborative designing
  - The following table shows the information flows between designers providing different activities (A, B, C, D, E) in product designing.
    - Construct a cooperation graph corresponding to all information flows.
    - Identify groups of interconnected activities (calculate the strong relation matrix and construct the corresponding graph with the components of the strong relation).

	A	B	C	D	E
A	0	1	1	0	0
B	1	0	1	1	0
C	0	0	0	1	0
D	0	0	0	0	1
E	0	0	1	0	0

Examples for MPLS exercises

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Placement of facilities

- Example 8 – Break-even point analysis
  - A company considers placement of facility. There are three locations at disposal – A, B and C. The following table shows fixed costs FC and variable costs VC connected with the production quantity q on facility in a location.
    - Decide where the facility will be placed at a production quantity of 600 units.
    - If you do not know the production quantity, determine the interval over which the production volume must vary for the location to be optimal in terms of total cost.

Location	FC	VC
A	200	$15q$
B	300	$10q$
C	170	$20q$

Examples for MPLS exercises

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Placement of facilities

- Example 9 – Multiple criteria evaluation of alternatives
  - The company wants to expand its business to another country. It is considering building a new facility in one of three countries (A, B, C). It evaluates each location according to the availability of materials, transport, climate and the level of taxes, with importance expressed as weights of 0.3, 0.4, 0.1 and 0.2. The table shows the evaluation on a scale of 0 to 100 of the options considered under each criterion. Decide in which country the new facility will be located. Use the Weighted sum approach.

Location	Material	Transport	Climate	Taxes
A	90	35	40	60
B	40	80	50	70
C	50	10	85	50

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Placement of facilities

- Example 10 – Method of gravity center
  - The company is deciding where to locate the facility, whose output will be distributed to five customers. The following table shows the coordinates of the customers and the goods quantity to be received. Find the optimal location (coordinates) of the new facility using both the simple and weighted center of gravity methods.

Location	x_i	y_i	q_i
A	2	2	50
B	5	6	100
C	3	4	30
D	8	3	60
E	5	1	70

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Placement of facilities

- Example 11 – Facility location problem (modified transportation problem)
  - The company currently has two warehouses (S1 and S2). The capacity of these warehouses is no longer sufficient to cover deliveries from two sources (D1 and D2). The company therefore needs to build another warehouse. The decision is between locations A and B, where the criterion under consideration is the total cost of transporting supplies from both sources. The unit transport costs (in thousands of CZK per tonne), the monthly warehouse capacities and required delivery quantities (both in tonnes) are given in the table below.
    - Find the optimal delivery for both options separately and decide which warehouse will be selected.
    - Formulate one mathematical model that considers both options simultaneously and finds the optimal delivery (only one of warehouses A and B can be selected).
    - How would the optimal solution change if the company rents warehouses A and B for monthly payments of 150 and 100 thousands of CZK?

	S1	S2	A	B	a_i
D1	5	6	3	4	100
D2	4	3	7	6	100
b_j	90	70	(40)	(40)	

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Placement of facilities

- Example 12 – Facility location problem (quadratic assignment problem)
  - The company intends to establish 5 warehouses in 5 cities. In the first table, distances (in km) between cities are given. The second table shows a number of necessary travels between warehouses within 1 month. The objective is to allocate the warehouses minimizing total travelling cost.

Distance	City1	City2	City3	City4	City5
City1	0	50	60	130	100
City2	50	0	70	150	120
City3	60	70	0	80	40
City4	130	150	80	0	50
City5	100	120	40	50	0

Travels	WH1	WH2	WH3	WH4	WH5
WH1	0	10	15	12	8
WH2	9	0	18	16	10
WH3	20	8	0	10	12
WH4	10	15	11	0	22
WH5	17	12	9	11	0

Examples for MPLS exercises

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Product schedule

- Example 13 – Feasible schedule
  - On a production line with 3 workplaces, where the product is produced, 10 operations need to be scheduled with the durations (in min) shown in the table. There are no precedence relations between operations. Takt time has been set to 10 min.

i	1	2	3	4	5	6	7	8	9	10
t_i	5	3	1	2	3	2	4	3	1	4

    - What is the product workload?
    - Decide, whether the following schedule is feasible:

$$\boldsymbol{A}_1 = \{1, 2, 4\},$$
$$\boldsymbol{A}_2 = \{3, 5, 6, 8\},$$
$$\boldsymbol{A}_3 = \{7, 9, 10\}.$$

    - Determine a number of idle minutes on the production line.
    - What is the production line efficiency?

Examples for MPLS exercises

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Product schedule

- Example 14 – Minimization of a number of workplaces
  - The company wants to schedule a product on the line, the production consists of 10 operations with durations (in min) given in the table(in min. There are no technological interdependencies between the operations, i.e. no precedence relation is defined. The takt time is set to 10 min.

i	1	2	3	4	5	6	7	8	9	10
t_i	6	3	4	5	2	2	4	3	7	2

  - What is a product workload?
  - Determine minimal and maximal number of workplaces on production line and find any feasible schedule.
  - For found schedule, determine a number of idle minutes on the whole production line and calculate its efficiency.

Examples for MPLS exercises

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Product schedule

- Example 15 – Minimization of takt time
  - The company wants to schedule a product with 10 operations on 5 workplaces. Their durations (in min) are given in the table. There is no precedence relation between the operations.

i	1	2	3	4	5	6	7	8	9	10
t_i	6	4	3	7	3	8	4	4	2	3

    - What is the theoretically achievable minimum takt time?
    - Find the minimal value of takt time.
    - What is the production line efficiency?

Examples for MPLS exercises

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Product schedule

- Example 16 – Determination of takt time for required daily production size
  - The company plans to achieve the daily production size Q=50 units in the working period of length T=8 hours. Production of the product consists of 10 operations with durations (in min) given in the table.

i	1	2	3	4	5	6	7	8	9	10
t_i	5	6	7	3	4	7	5	3	2	4

  - What is the highest possible takt time at which the desired daily production can still be achieved?
  - Is it possible to find a feasible schedule for this takt time with the existing 6 workplaces? What will be the efficiency of the whole production line?
  - What can be the minimum takt time without increasing the number of workplaces? What is the efficiency of the line in this case?

Examples for MPLS exercises

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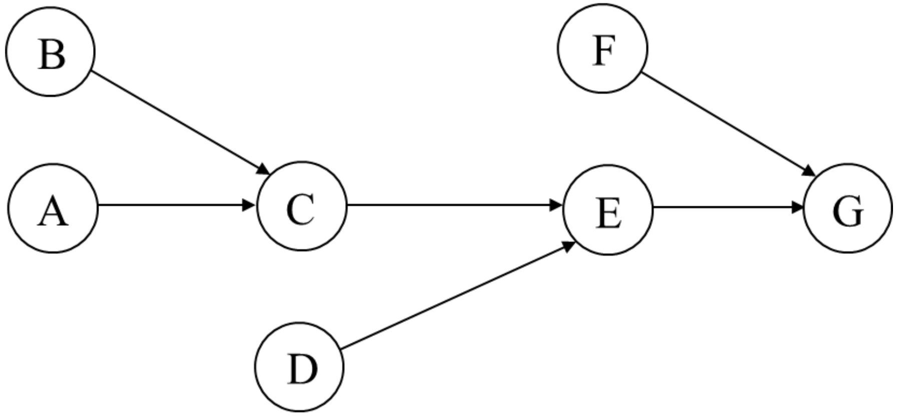
Product schedule

- Example 17 – Heuristic methods for scheduling operations
  - The company plans to produce 50 units of product per day, which production consists of 7 operations. All precedence relations for them are illustrated by the directed graph in the figure. In the table, the durations of operations are given. The length of the daily working period is 8 hours. Schedule all operations on workplaces using all 4 heuristics.



Operation	A	B	C	D	E	F	G
t_i	4	2	3	6	2	4	5

Examples for MPLS exercises

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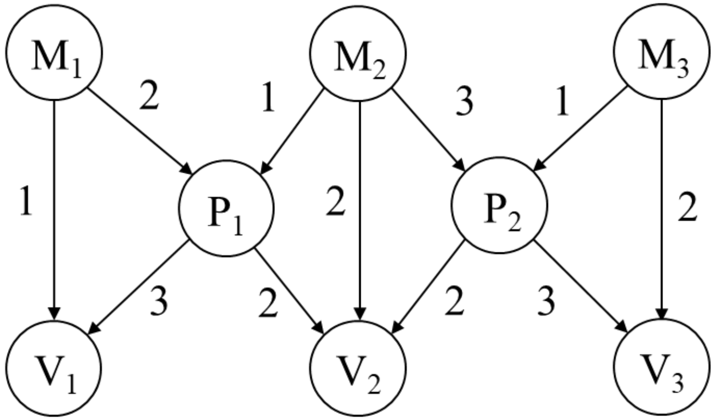
Production planning

- Example 18 – Multi-level planning
  - The company produces three final products V_1, V_2 and V_3 in 40, 20 and 30 units. Three types of materials M_1, M_2, M_3 and two semi-finished products P_1 and P_2 are used. Product structure is expressed by Gozinto graph. Calculate all production input and output values:
    - Use the Leontief model.
    - Use the optimization model.



Examples for MPLS exercises

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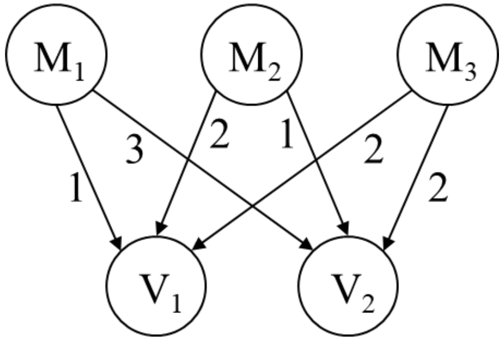
Production planning

- Example 19 – Multi-level planning (MRP)
  - For 1 unit of product V_1, 1 unit of material M_1, 2 units of material M_2 and 2 units of material M_3 are required. For the production of 1 unit of product V_2, 3 units of material M_1, 1 unit of material M_2 and 2 units of material M_3 (see the figure). The table shows the initial inventory level of materials and final products. There are their delivery times given in weeks. Products can be produced as soon as all the material for them is available. The company has to deliver 250 units of product V_1 in 7th week and 180 units of product V_2 in 8th week, and 200 units od product V_1 and 160 units of product V_2 in 17th week. The objective is to plan the material requirements for the whole period so that the demand for both products is met.



Item	Initial inventory	Delivery time
M_1	20	1
M_2	60	2
M_3	60	2
V_1	100	1
V_2	120	2

Examples for MPLS exercises

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Production scheduling

- Example 20 – Models $1||F_{\max}$ and $1||F_{\max}(w)$
  - Consider 7 jobs, for which you know their processing time (in hours), their due time (in hour) and their relative importance in terms of weight (see table). The ready times for all jobs are zero.
    - Find the optimal schedule for model $1||F_{\max}$. Determine the total tardiness of all jobs.
    - Try to find the optimal schedule that would meet the due times of all jobs.
    - Find the optimal schedule for model $1||F_{\max}(w)$.

Job	t_j	d_j	w_j
D_1	4	7	0,15
D_2	2	10	0,11
D_3	1	18	0,16
D_4	3	6	0,10
D_5	5	15	0,08
D_6	2	3	0,17
D_7	1	20	0,23

Examples for MPLS exercises

Production scheduling

- Example 21 – Models $M1||\bar{F}$ and $1||\bar{F}(w)$
  - Find the optimal schedule for model $1||\bar{F}$.
  - Find the optimal schedule for model $1||\bar{F}(w)$.

Job	t_j	d_j	w_j
D_1	4	7	0,15
D_2	2	10	0,11
D_3	1	18	0,16
D_4	3	6	0,10
D_5	5	15	0,08
D_6	2	3	0,17
D_7	1	20	0,23

Examples for MPLS exercises

Production scheduling

- Example 22 – Models $1||L_{\max}$, $1||T_{\max}$, $1||L_{\max}(w)$ and $1||T_{\max}(w)$
  - Find the optimal schedule for model $1||L_{\max}$ $(1||L_{\max})$. Determine the total tardiness of all jobs.
  - Find the optimal schedule for model $1||T_{\max}(w)$ using Lawler‘s algorithm.

Job	t_j	d_j	w_j
D_1	4	7	0,15
D_2	2	10	0,11
D_3	1	18	0,16
D_4	3	6	0,10
D_5	5	15	0,08
D_6	2	3	0,17
D_7	1	20	0,23

Examples for MPLS exercises

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Production scheduling

- Example 23 – Models $1||\bar{L}$ and $1||\bar{L}(w)$
  - Find the optimal schedule for model $1||\bar{L}$.
  - Find the optimal schedule for model $1||\bar{L}(w)$.

Job	t_j	d_j	w_j
D_1	4	7	0,15
D_2	2	10	0,11
D_3	1	18	0,16
D_4	3	6	0,10
D_5	5	15	0,08
D_6	2	3	0,17
D_7	1	20	0,23

Examples for MPLS exercises

Production scheduling

- Example 24 – Model $1||N$
  - Find the optimal schedule for model $1||N$ using Moore's algorithm.

Job	t_j	d_j	w_j
D_1	4	7	0,15
D_2	2	10	0,11
D_3	1	18	0,16
D_4	3	6	0,10
D_5	5	15	0,08
D_6	2	3	0,17
D_7	1	20	0,23

Examples for MPLS exercises

Production scheduling

- Example 25 – Model $1|s_{jk}|C_{\max}$
  - The company wants to schedule 7 jobs on 1 machine so that they are completed in minimum time. The table shows the job processing times and the setup times between jobs (all values are in minutes).
    - Find a schedule using the nearest neighbor algorithm for the travelling salesman problem.
    - Find the optimal schedule using the mathematical model for the travelling salesman problem.

	D_1	D_2	D_3	D_4	D_5	D_6	D_7	t_j
D_1	0	120	150	110	95	80	95	120
D_2	120	0	130	80	95	105	120	110
D_3	130	85	0	90	105	110	100	105
D_4	90	140	135	0	120	110	105	90
D_5	85	70	95	80	0	90	100	85
D_6	105	135	120	125	160	0	155	115
D_7	110	120	105	95	120	115	0	95

Examples for MPLS exercises

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Production scheduling

- Example 26 – Models $P_m|prmp|F_{\max}$ and $P_m||F_{\max}$
  - The objective is to schedule 12 jobs on 4 machines so that they are executed in minimum time. The table shows the processing times of the jobs (in min). The operations that make up each job have a duration of 1 min.

	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9	D_{10}	D_{11}	D_{12}
t_j	3	5	2	4	8	6	5	5	4	3	7	3

  - Find the optimal schedule with the possibility to interrupt the processing of jobs.
  - Find the optimal schedule without interruptions.

Examples for MPLS exercises

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Production scheduling

- Example 27 – Model $P_m||\bar{F}$
  - Find the optimal schedule without interruptions of processing of the jobs..

	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9	D_{10}	D_{11}	D_{12}
t_j	3	5	2	4	8	6	5	5	4	3	7	3

Examples for MPLS exercises

Production scheduling

- Example 28 – Model $F_m||C_{\max}$
  - There are 10 jobs given, which must be scheduler to 2 serial machines. The table shows the processing times of the jobs (in min) on both machines. The objective is to minimize the completion time of all jobs.

	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9	D_{10}
t_{1j}	1	6	3	3	5	7	6	2	3	5
t_{2j}	3	5	1	5	6	4	5	4	5	4

Examples for MPLS exercises

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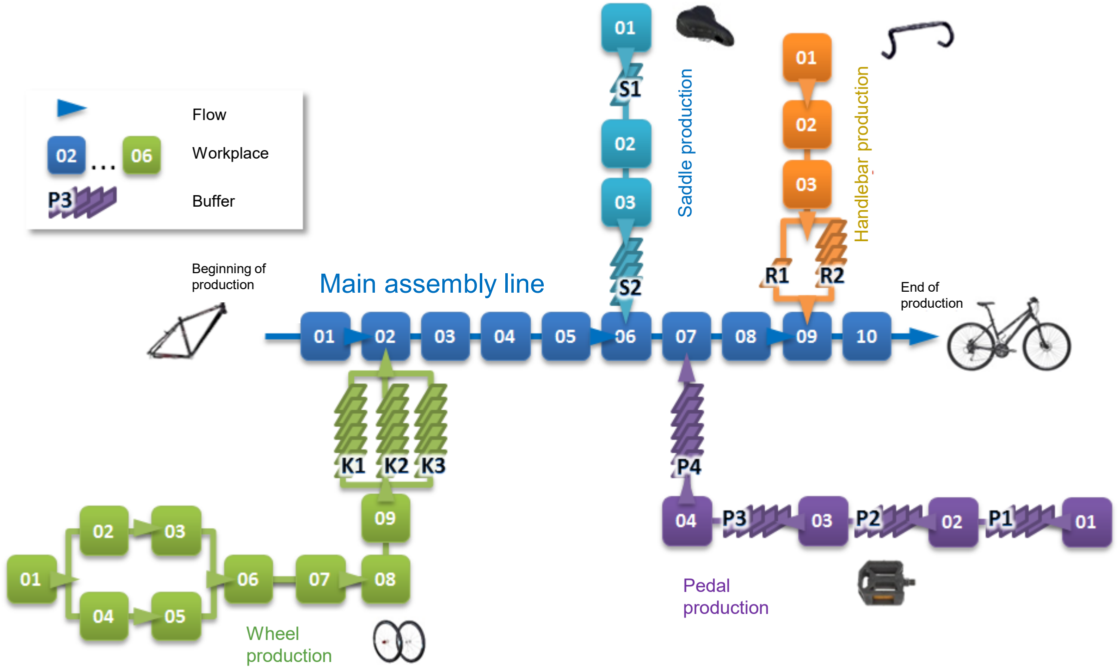
Počítačová simulace

- Example 29 – Generation of the values of random variables
  - Generate 1000 values of random variable X with the uniform probability distribution on interval (100, 200) and make a graph of the frequencies of values at predefined intervals..

Examples for MPLS exercises

Počítačová simulace

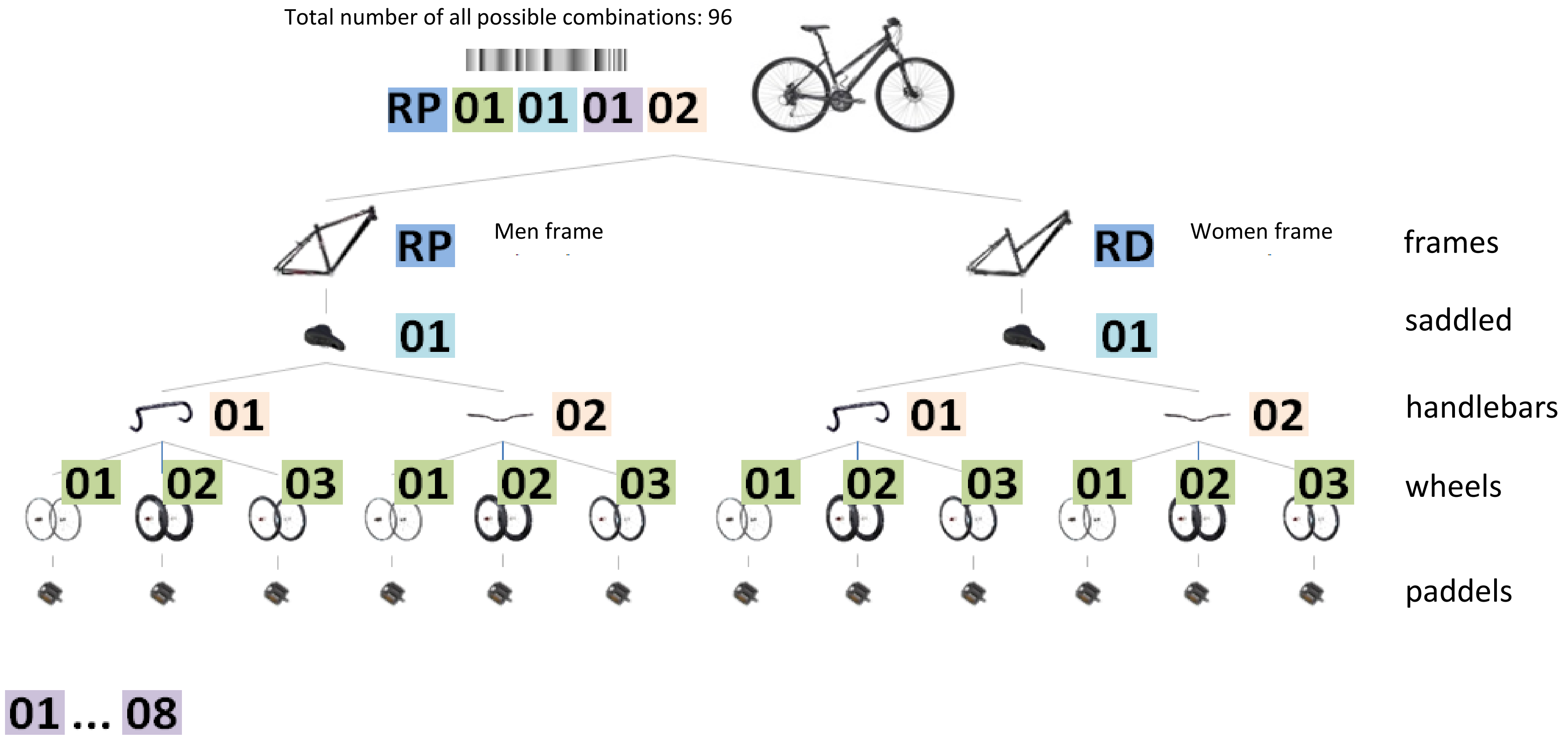
- Example 30 – Production of bicycles
  - The production system consists of one main assembly line and four production sections (production of wheels, saddles, pedals and handlebars). The production sections are connected to the main assembly line by buffers. The FIFO principle is followed in them. The shift calendar is identical on all lines.



Examples for MPLS exercises

Počítačová simulace

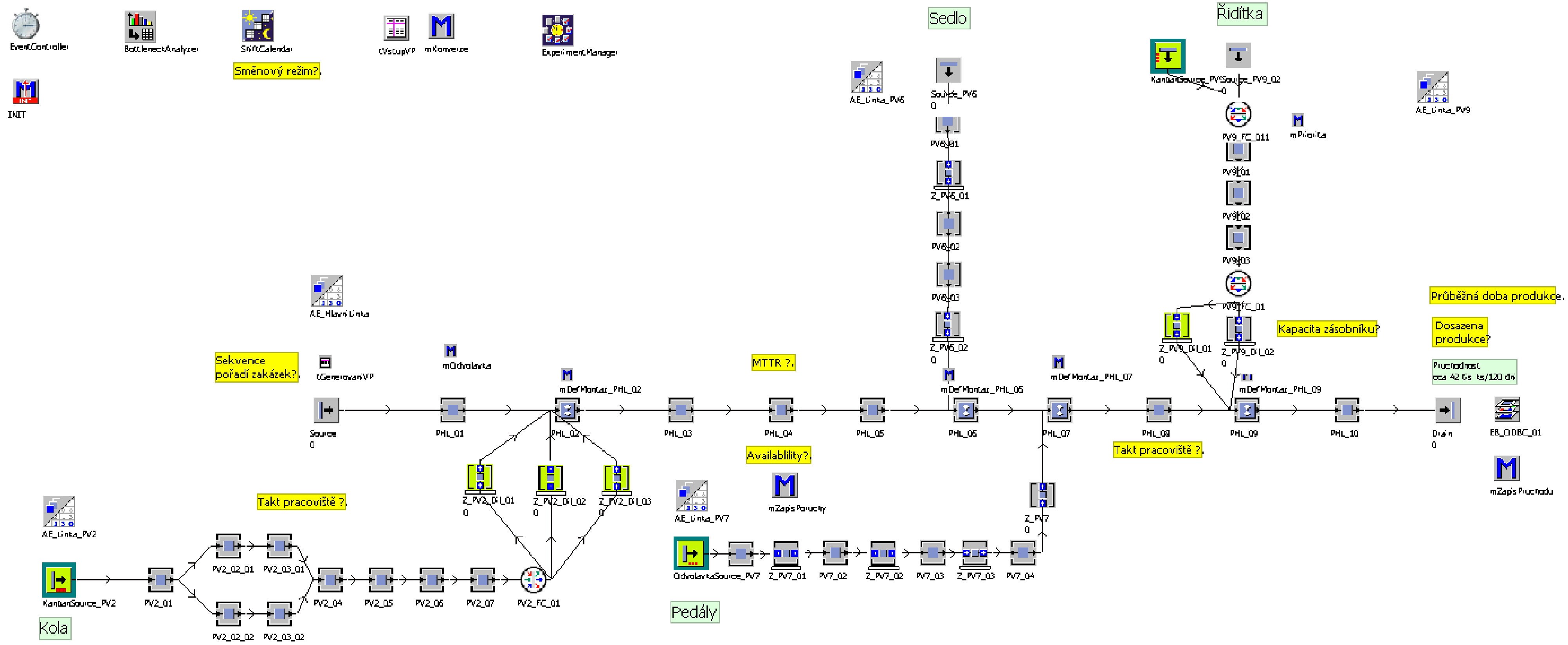
- Example 30 – Production of bicycles – continued



Examples for MPLS exercises

Počítačová simulace

- Example 30 – Production of bicycles – continued



Examples for MPLS exercises

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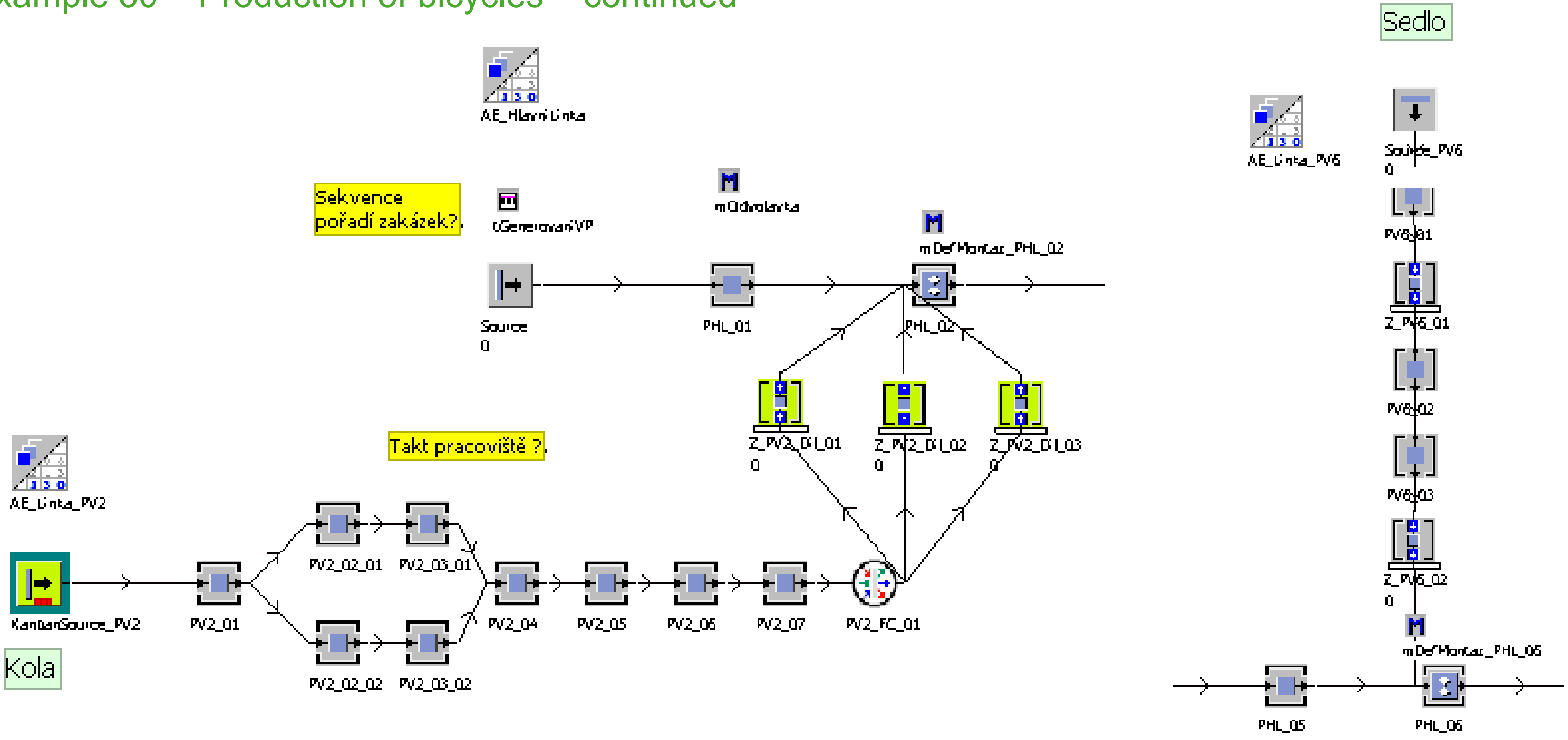
Počítačová simulace

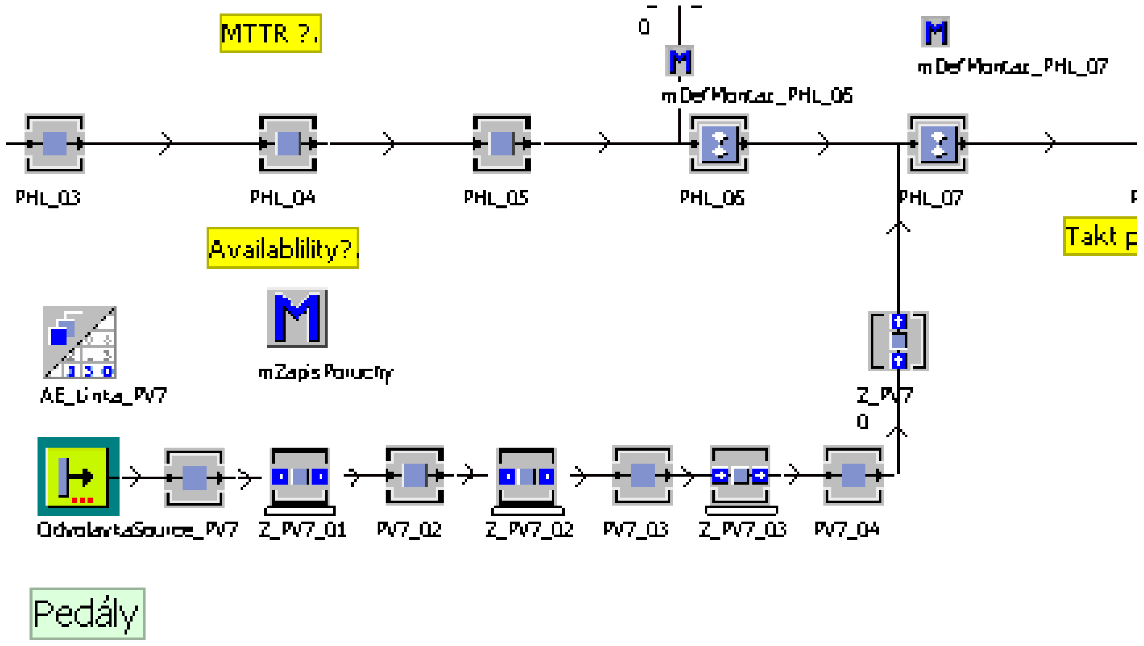
- Example 30 – Production of bicycles – continued

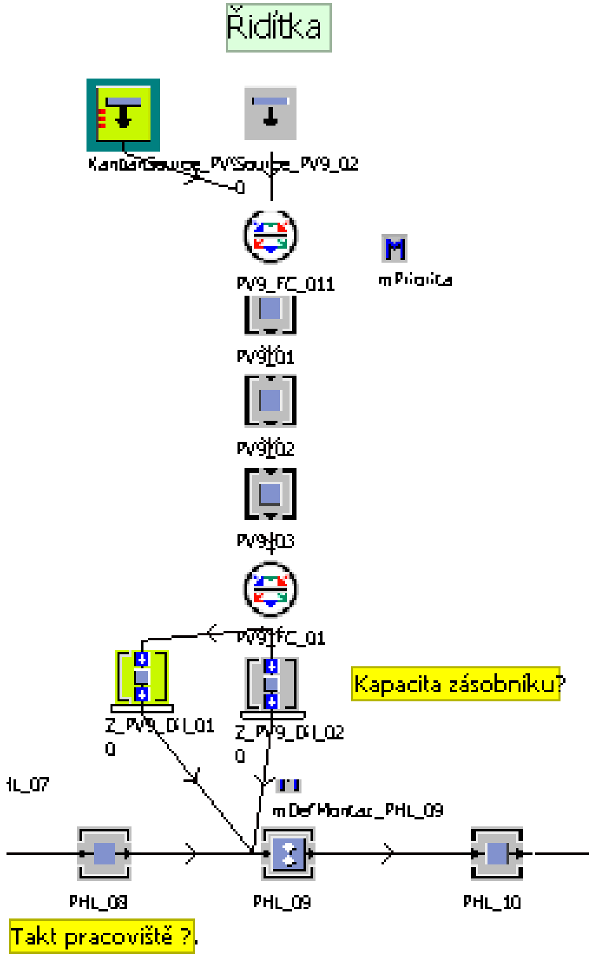


Examples for MPLS exercises

Počítačová simulace

- Example 30 – Production of bicycles – continued

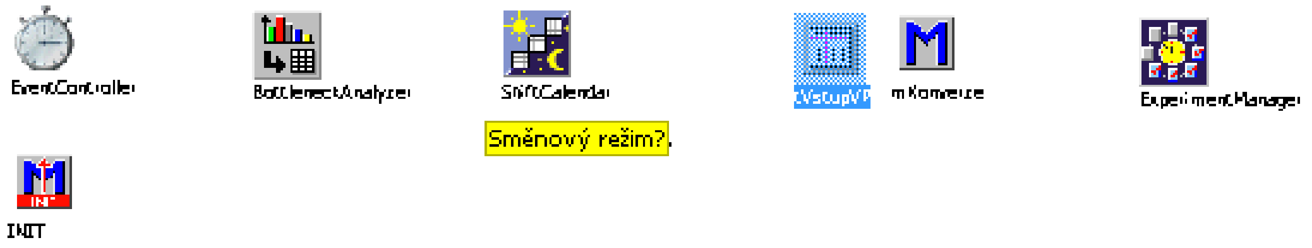




Examples for MPLS exercises

Počítačová simulace

- Example 30 – Production of bicycles – continued



Examples for MPLS exercises

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Počítačová simulace

- Example 30 – Production of bicycles – continued
  - Data analysis
    - Mean of daily production quantity (sheet „10_Vystup“).
    - Production program (sheet „01_Vstup“).
    - Continuous workload time of the main line (sheets „01_Vstup“ and „10_Vystup“).
    - Shift and pauses scheme (sheet „10_Vystup“).
    - Capacity of buffer R2 (sheet „R2“).
    - Takt time on workplace 03 of peddals production line (shett „Takt_03“).
    - Takt time on workplace 08 of main assembly line (sheet „Takt_08“).
    - Mean time ti repair (MTTR) on workplace 04 of main assembly line (shett „MTTR“).
    - Availability of workplace 04 of main assembly line (sheets „MTTR“ and „10_Vystup“).

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Thank you for attention

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