4EK605 Combinatorial Optimization Exercises

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Exercise 1

Example 1

Production planning with semi-finished products

Firm produces products P_1, P_2 and P_3.
To produce 1 unit of product P_1, the firm uses 3 kg of material.
To produce 1 unit of product P_2, the firm uses 2 kg of material and 1 unit of product P_1.
To produce 1 unit of product P_3, the firm uses 2 kg of material, 2 units of product P_1 and 1 unit of product P_2.
There are 1000 kg of material available.
Products P_1 and P_2 that are used as semi-finished products can also be sold themselves.
Prices of goods P_1, P_2 and P_3 are 5, 10 and 30 €. The objective is to maximize total revenues from products sold.

Formulate a mathematical model of the problem and solve it in MPL for Windows.

Exercise 1

Example 2

Cutting Stock Problem

Firm produces garden laths fence. There are only standard laths 200 cm long at disposal at storehouse. To produce a fence, firm needs exactly 1200 laths 80 cm long, 3100 laths 50 cm long and 2100 laths 30 cm long.
You have to design a cutting plan to minimize total amount of laths 200 cm long.

Formulate a mathematical model of the problem and solve it in MPL for Windows.

Exercise 2

Example 3

Knapsack Problem

There are 5 projects characterized by the investment cost and return. The budget 50 000 € is available to select such projects that assure the highest total return.

	P1	P2	P3	P4	P5
Cost	12 000	10 000	15 000	18 000	16 000
Return	20 000	18 000	22 000	26 000	21 000

Exercise 2

Example 4

Perfect Matching Problem

Ten students go for a school trip. To assign them to double rooms, they were asked to express their preferences (see the table, 0-min, 10-max). For $i < j$, the value c_{ij} is the preference value expressing student i wants to be in the room with student j, for $i > j$, the value c_{ij} is the preference value expressing student j wants to be in the room with student i. Will you assign students to rooms to maximize total happiness of the group.

Exercise 2

Example 4

Perfect Matching Problem

Pref	1	2	3	4	5	6	7	8	9	10
1	0	7	6	2	4	7	4	1	8	3
2	1	0	3	1	10	5	2	9	4	2
3	10	1	0	5	6	1	8	2	7	4
4	1	8	4	0	10	7	5	4	2	7
5	8	7	3	5	0	2	1	5	2	9
6	2	2	3	7	8	0	8	2	1	5
7	1	7	6	1	7	7	0	8	1	5
8	6	8	1	1	10	8	1	0	4	7
9	4	1	2	2	8	1	7	5	0	2
10	1	5	4	3	9	7	1	4	6	0

Example 5

Linear Assignment Problem

Relay race for 5-member teams is organized. A member of each team will be competing in one discipline. You are going to build a strongest team. In the table, the seasonal best performances (in minutes) of candidates are given.

SB	Run	Swim	Bike	Inline	Ski
Mike	75	25	202	130	165
Jack	87	24	198	127	173
Peter	68	19	195	121	164
Sean	91	20	207	122	182
Paul	80	28	215	125	172
Simon	78	22	197	125	180
Tom	75	25	205	127	178
David	81	23	211	131	165

Exercise 3

Example 6

Bottleneck Assignment Problem

The project consists of 5 independent parts. In the company 5 departments can manage the parts individually. Historical data shows average times (in days) departments finished similar tasks (see the table). N.A. represents the fact a department did not work on such task in the past. The company wants to finish the whole project as soon as possible.

Time	Part1	Part2	Part3	Part4	Part5
Dept1	25	15	N.A.	17	25
Dept2	22	N.A.	22	20	22
Dept3	20	18	25	16	23
Dept4	N.A.	20	30	21	28
Dept5	27	19	27	18	N.A.

Exercise 4

Example 7

Quadratic Assignment Problem

The company intends to establish 5 warehouses in 5 cities. In the first table, distances (in km) between cities are given. The second table shows a number of necessary travels between warehouses within 1 month. The objective is to allocate the warehouses minimizing total travelling cost.

Example 7

Quadratic Assignment Problem

Distance	City1	City2	City3	City4	City5
City1	0	50	60	130	100
City2	50	0	70	150	120
City3	60	70	0	80	40
City4	130	150	80	0	50
City5	100	120	40	50	0

Travels	WH1	WH2	WH3	WH4	WH5
WH1	0	10	15	12	8
WH2	9	0	18	16	10
WH3	20	8	0	10	12
WH4	10	15	11	0	22
WH5	17	12	9	11	0

Example 8

Facility Location Problem

The company can use 7 potential warehouses for its 5 subsidiaries. In the table, monthly requirements of subsidiaries and monthly capacities of warehouses are given (in thousands of tons).
If a warehouse is used, the company must pay monthly rent (in thousands €). In addition, unit transportation cost (in € per ton) is calculated for each pair of the warehouse and subsidiary. Will you decide which warehouse to use and what amounts of material to transport between warehouses and subsidiaries. The objective is to minimize total monthly cost.

Example 8

Facility Location Problem

FLP	SD1	SD2	SD3	SD4	SD5	Cap	Rent
WH1	10	15	20	12	8	20	10
WH2	7	10	15	22	13	25	12
WH3	20	13	10	11	9	15	8
WH4	15	12	21	18	16	18	9
WH5	11	22	12	10	15	22	11
WH6	9	13	11	18	22	30	13
WH7	18	10	15	7	9	23	11
Req	25	22	17	22	15		

Example 9

Bin Packing Problem

Products must be transported to the client using identical containers. In the table, a unit weight of each product type (in kg) and a number of them to transport are given. The weight capacity of the container is 500 kg. The objective is to minimize a number of used containers.

BinPacking	Weight	Number
Product1	20	13
Product2	22	15
Product3	18	25
Product4	15	30
Product5	21	18
Product6	16	35

Exercise 5

Example 10

Maximum Flow Problem

Will you find the maximum flow from node 1 to node 6 for a graph given by the following table.

Arc	Capacity	Arc	Capacity
(1,2)	10	(3,5)	7
(1,3)	10	(3,6)	5
(1,4)	12	(4,3)	3
(2,5)	11	(4,6)	9
(3,4)	3	(5,6)	18

Exercise 5

Example 11

Minimum-Cost Flow Problem

Will you find the flow (from 1 to 6) of value 25 with the minimal total cost. In the table, capacity and unit cost for each arc are given.

Arc	Capacity	Cost	Arc	Capacity	Cost
(1,2)	10	5	(3,5)	7	6
(1,3)	10	10	(3,6)	5	9
(1,4)	12	20	(4,3)	3	12
(2,5)	11	11	(4,6)	9	17
(3,4)	3	12	(5,6)	18	8

Example 12

Maximum Flow Cost-Limited Problem

Let 700 € be the budget for the flow. Will you find the maximum flow from 1 to 6 respecting this restriction.

Arc	Capacity	Cost	Arc	Capacity	Cost
(1,2)	10	5	(3,5)	7	6
(1,3)	10	10	(3,6)	5	9
(1,4)	12	20	(4,3)	3	12
(2,5)	11	11	(4,6)	9	17
(3,4)	3	12	(5,6)	18	8

Exercise 5

Example 13

Transshipment Problem

It is necessary to transport empty containers from sources to destinations. In the graph, nodes 1 and 3 are sources with supply 15 and 10 containers, nodes 4 and 6 are destinations with demand 5 and 20 containers. The objective is to minimize total cost.

Arc	Capacity	Cost	Arc	Capacity	Cost
(1,2)	10	5	(3,5)	7	6
(1,3)	10	10	(3,6)	5	9
(1,4)	12	20	(4,3)	3	12
(2,5)	11	11	(4,6)	9	17
(3,4)	3	12	(5,6)	18	8

Exercise 5

Example 14

Minimal Spanning Tree

The company has to install 6 information boards in the city park. They must be connected by cable leading under pavements. Distances (in ten meters) between boards can be found in the table. If there is no pavement between a pair of boards, prohibitive value 100 is set. The objective is to minimize total cost both on excavation work and on cable itself.

Boards	1	2	3	4	5	6
1	0	6	5	100	100	100
2	6	0	7	2	4	100
3	5	7	0	6	100	8
4	100	2	6	0	3	4
5	100	4	100	3	0	5
6	100	100	8	4	5	0

Exercise 5

Example 15

Minimal Steiner Tree

Three users (nodes 2, 3 and 4) must be connected to the transmitter (node 1) either directly or through two transfer stations (nodes 5 and 6). In the table, cost values (in thousands € per month) for possible connections are given. Use of transfer stations is charged 30 and 20 thousands € per month. Will you find the optimal connection.

Arc	Cost	Arc	Cost
(2,1)	15	(4,5)	9
(2,5)	3	(4,6)	6
(3,1)	18	(5,1)	7
(3,5)	4	(6,1)	12
(3,6)	7		

Example 16

Travelling Salesman Problem

A sales representative of the brewery located in Velvary must visit 7 pubs in 7 cities. In the following table, distances (in km) correspond to direct links (roads) between cities. A dash indicates there is no direct road between cities. The objective is to visit all pubs minimizing total length of the tour.

	Velv	Kra	Lib	Sla	Zlo	Vra	Bri	Velt
Velvary	0	8	-	13	10	-	12	9
Kralupy	8	0	6	16	-	-	-	4
Libcice	-	6	0	-	-	-	-	-
Slany	13	16	-	0	7	-	-	-
Zlonice	10	-	-	7	0	7	13	-
Vrany	-	-	-	-	7	0	15	-
Briza	12	-	-	-	13	15	0	13
Veltrusy	9	4	-	-	-	-	13	0

Example 16

Travelling Salesman Problem

The following table contains distances between all pairs of cities.

Distance	Velv	Kra	Lib	Sla	Zlo	Vra	Bri	Velt
Velvary	0	8	14	13	10	17	12	9
Kralupy	8	0	6	16	18	25	17	4
Libcice	14	6	0	22	24	31	23	10
Slany	13	16	22	0	7	14	20	20
Zlonice	10	18	24	7	0	7	13	19
Vrany	17	25	31	14	7	0	15	26
Briza	12	17	23	20	13	15	0	13
Veltrusy	9	4	10	20	19	26	13	0

Example 17

Vehicle Routing Problem

The sales representative of the brewery (see Example 16) has arranged advantageous contracts. Pubs will take barrels of beer in quantities given in the following table. For delivery, a vehicle with the capacity of 50 barrels will be used. The objective is to satisfy all requirements minimizing total length of the vehicle tours.

	Requirement
Velvary	0
Kralupy	18
Libcice	10
Slany	15
Zlonice	12
Vrany	10
Briza	8
Veltrusy	11

Exercise 6

Example 18

Undirected Chinese Postman Problem

At Halloween, trik-or-treating children want to visit all houses in neighborhood (see the figure). The lengths of streets (in meters), they must go through, are given in the table. Will you plan a tour for children to minimize the total distance.

Exercise 6

Example 18

Undirected Chinese Postman Problem

Arc	Length	Arc	Length
(1,2)	210	(6,7)	80
(1,9)	160	(6,11)	150
(2,3)	140	(7,8)	80
(2,5)	80	(7,9)	110
(3,4)	40	(9,10)	160
(3,5)	210	(10,11)	130
(4,6)	310	(10,12)	190
(5,6)	70	(11,12)	150

Example 19

Undirected Chinese Postman Problem

A vehicle, taking photos for Street View application, must visit positions along the given streets in Brno city (see the figure). The lengths of streets (in meters) are given in the table. Will you plan a tour of the vehicle to minimize its total travel distance.

Exercise 6

Example 19

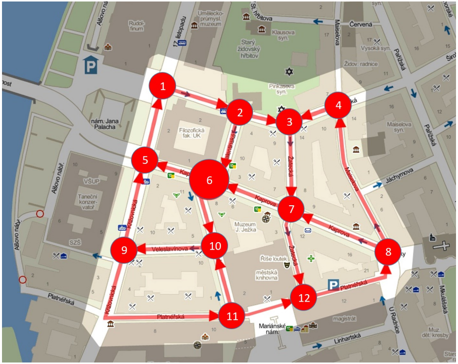
Undirected Chinese Postman Problem

Length	1	2	3	4	5	6	7	8	9	10
1	-	91	-	226	111	-	-	-	-	-
2	91	-	90	158	186	-	-	-	-	-
3	-	90	-	-	451	68	-	-	-	158
4	226	158	-	-	-	-	189	-	-	-
5	111	186	451	-	-	-	-	-	-	-
6	-	-	68	-	-	-	56	-	157	-
7	-	-	-	189	-	56	-	170	-	-
8	-	-	-	-	-	-	170	-	91	358
9	-	-	-	-	-	157	-	91	-	72
10	-	-	158	-	-	-	-	358	72	-

Example 20

Directed Chinese Postman Problem

A vehicle collecting garbage from bins must go through one-way streets in a district of Prague (see the figure). The lengths of streets (in meters) are given in the table. The objective is to minimize the total distance the vehicle travels.



Exercise 6

Example 20

Directed Chinese Postman Problem

Arc	Length	Arc	Length
(1,2)	82	(7,12)	93
(2,3)	53	(8,4)	162
(2,6)	78	(8,7)	111
(3,7)	93	(9,5)	93
(4,3)	56	(9,11)	200
(5,1)	78	(10,9)	96
(6,5)	80	(11,10)	73
(6,10)	76	(11,12)	76
(7,6)	78	(12,8)	111